JEE ADVANCED-2010

PAPER-1

IMPORTANT INSTRUCTIONS

A. General:

1. This question paper contains 32 pages having 57 questions.
2. The **question paper CODE** is printed on the right hand top corner of this sheet and on the back page (page no. 32) of this booklet.
3. No additional sheets will be provided for rough work.
4. Blank papers, clipboards, log tables, slide rules, calculators, cellular phones, pagers, and electronic gadgets in any from are not allowed.
5. Log and Antilog tables are given in page numbers 30 and 31 respectively.
6. The answer sheet, a machine-gradable. Objective Response sheet **(ORS)**, is provided separately.
7. DO NOT TAMPER WITH /MUTILATE THE ORS OR THE BOOKLET.
8. Do not break the seals of the question–paper booklet before instructed to do so by the invigilation.

B. Filling the Right Part of the ORS:

9. The ORS also has a **CODES** printed on its lower and upper parts.
10. Make sure the **CODE** on the **ORS** is the same its that on this booklet. If the Codes do not match, ask **for a change of the Booklet**.
11. Write your Registration No., Name and Name of centre and sign with pen in appropriate boxes. Do not the boxes write these anywhere else.
12. Darken the appropriate bubbles under each digit of your Registration No. with HB Pencil.

C. Question paper format and Marking scheme:

13. The question paper consists of **3 parts** (Chemistry, Mathematics and Physics). Each part consists of four sections.
14. For each question in **Section I:** you will be awarded **5 marks if you darken only the bubble** corresponding to the correct answer and **zero mark** if no bubbles on darkened. In all other cases, **minus two (-2) mark** will be awarded.
15. For each question in **Section II:** you will be awarded 3 marks if you darken the bubble corresponding to the correct answer and **zero mark** if no bubbles are darkened. No negative marks will be awarded for incorrect answers in this section.
16. For each question in **Section III:** you will be **awarded 3 marks** if you darken **only** the bubble corresponding to the correct answer and **zero marks** if bubbles are darkened. In all other cases, **minus one (-1)** mark will be awarded.
17. For each question in **Section IV**, you will be awarded **2 marks** for each now in which your darkened the **bubbles(s)** corresponding to the correct answer. Thus each question in this section carries a maximum of **8 marks**. There is no negative marks awarded for incorrect answer(s) in this Section.

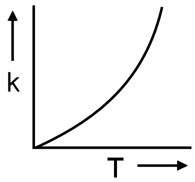
PART-A: CHEMISTRY

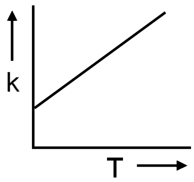
SECTION-I

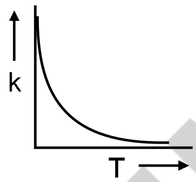
(Single Correct Choice Type)

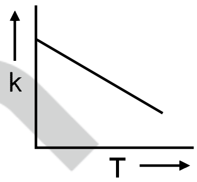
This section contains **8 multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

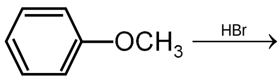
1. Plots showing the variation of the rate constant (k) with temperature (T) are given below. The plot that follows Arrhenius equation is

(A*) 

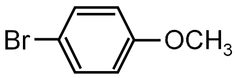
(B) 

(C) 

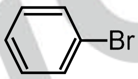
(D) 

2. In the reaction 

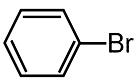
the products are

(A) 

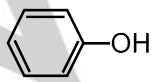
and H_2

(B) 

and CH_3Br

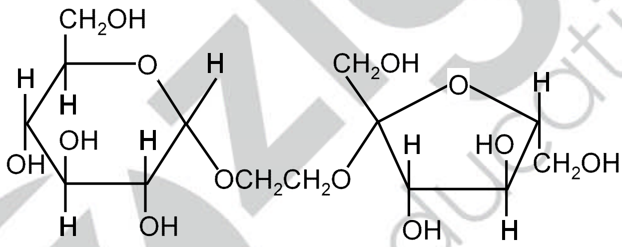
(C) 

and CH_3OH

(D*) 

and CH_3Br

3. The correct statement about the following disaccharide is



(a) **(b)**

(A*) Ring **(a)** is pyranose with α-glycosidic link.

(B) Ring **(a)** is furanose with α-glycosidic link

(C) Ring **(b)** is furanose with α-glycosidic link

(D) Ring **(b)** is pyranose with β-glycosidic link

4. The synthesis of 3-octyne is achieved by adding a bromoalkane into a mixture of sodium amide and an alkyne. The bromoalkane and alkyne respectively are

(A) $BrCH_2CH_2CH_2CH_2CH_3$ and $CH_3CH_2C{\equiv}CH$

(B) $BrCH_2CH_2CH_3$ and $CH_3CH_2CH_2C{\equiv}CH$

(C) $BrCH_2CH_2CH_2CH_2CH_3$ and $CH_3C{\equiv}CH$

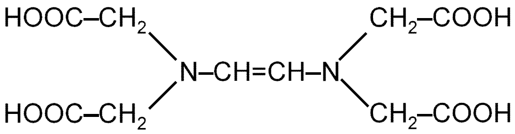
(D*) $BrCH_2CH_2CH_2CH_3$ and $CH_3CH_2C{\equiv}CH$

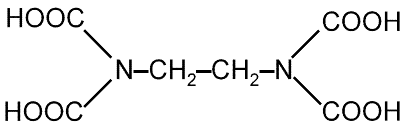
5. The ionization isomer of $[Cr(H_2O)_4Cl(NO_2)]Cl$ is

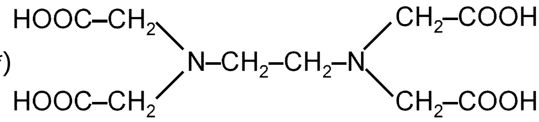
(A) $[Cr(H_2O)_4(O_2N)]Cl_2$ (B*) $[Cr(H_2O)_4Cl_2]\ (NO_2)$

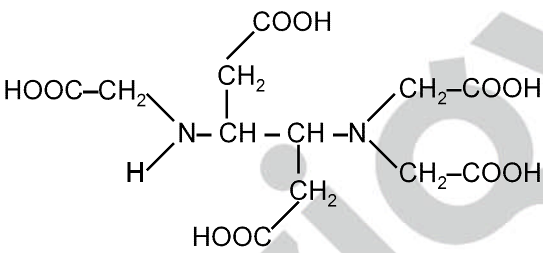
(C) $[Cr(H_2O)_4Cl(ONO)]Cl$ (D) $[Cr(H_2O)_4Cl_2(NO_2)]\cdot H_2O$

6. The correct structure of ethylenediaminetetraacetic acid (EDTA) is

(A) 

(B) 

(C*) 

(D) 

7. The bond energy **(**in **kcal mol^{-1})** of a C–C single bond is approximately

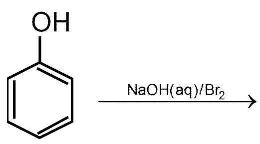
(A) 1 (B) 10 (C*) 100 (D) 1000

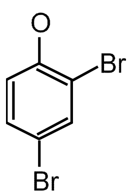
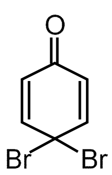
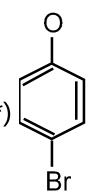
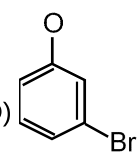
8. The species which by definition has **ZERO** standard molar enthalpy of formation at 298 K is

(A) Br_2 (g) (B*) Cl_2 (g) (C) H_2O (g) (D) CH_4 (g)

SECTION-II
(Multiple Correct Choice Type)

This section contains **5 multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE** may be correct.

9. In the reaction  the intermediate(s) is(are)

(A*)  (B*)  (C*)  (D) 

10. Among the following, the intensive property is (properties are)

(A*) molar conductivity (B*) electromotive force

(C) resistance (D) heat capacity

11. The reagent(s) used for softening the temporary hardness of water is(are)

(A) $Ca_3(PO_4)_2$ (B*) $Ca(OH)_2$ (C*) Na_2CO_3 (D*) NaOCl

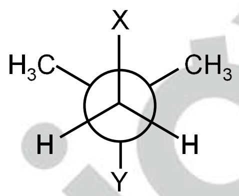
12. Aqueous solutions of HNO_3, KOH, CH_3COOH, and CH_3COONa of identical concentrations are provided. The pair(s) of solutions which form a buffer upon mixing is(are)

(A) HNO_3 and CH_3COOH (B) KOH and CH_3COONa

(C*) HNO_3 and CH_3COONa (D*) CH_3COOH and CH_3COONa

[Option 'C' is correct under certain condition]

13. In the Newman projection for 2,2-dimethylbutane



X and **Y** can respectively be

(A) H and H (B*) H and C_2H_5 (C) C_2H_5 and H (D*) CH_3 and CH_3

SECTION-III

(Paragraph Type)

This section contains **2 paragraphs**. Based upon the first paragraph **3 multiple choice questions** and based upon the second paragraph **2 multiple choice questions** have to be answered. Each of these questions has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Questions 14 to 16

Copper is the most noble of the first-row transition metals and occurs in small deposits in several countries. Ores of copper include chalcanthite ($CuSO_4 \cdot 5H_2O$), atacamite ($Cu_2Cl(OH)_3$), cuprite (Cu_2O), copper glance (Cu_2S) and malachite ($Cu_2(OH)_2CO_3$). However, 80% of the world copper production comes from the ore chalcopyrite ($CuFeS_2$). The extraction of copper from chalcopyrite involved partial roasting, removal of iron and self-reduction.

14 Partial roasting of chalcopyrite produces

(A*) Cu_2S and FeO (B) Cu_2O and FeO

(C) CuS and Fe_2O_3 (D) Cu_2O and Fe_2O_3

15. Iron is removed from chalcopyrite as

(A) FeO (B) FeS (C) Fe_2O_3 (D*) $FeSiO_3$

16. In self-reduction, the reducing species is

(A) S (B) O^{2-} (C*) S^{2-} (D) SO_2

Paragraph for Questions 17 to 18

The concentration of potassium ions inside a biological cell is at least twenty times higher than the outside. The resulting potential difference across the cell is important in several processes such as transmission of nerve impulses and maintaining the ion balance. A simple model for such a concentration cell involving a metal M is:

$M(s) \mid M^+(aq; 0.05\ molar) \mid\mid M^+ (aq; 1\ molar) \mid M(s)$

For the above electrolytic cell the magnitude of the cell potential $|E_{cell}| = 70$ mV.

17. For the above cell

(A) $E_{cell} < 0; \Delta G > 0$ (B*) $E_{cell} > 0; \Delta G < 0$

(C) $E_{cell} < 0 ; \Delta G° > 0$ (D) $E_{cell} > 0; \Delta G° < 0$

18. If the 0.05 molar solution of M^+ is replaced by a 0.0025 molar M^+ solution, then the magnitude of the cell potential would be

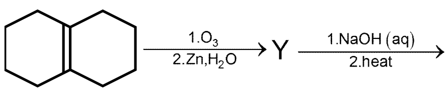
(A) 35 mV (B) 70 mV (C*) 140 mV (D) 700 mV

SECTION-IV

(Integer Type)

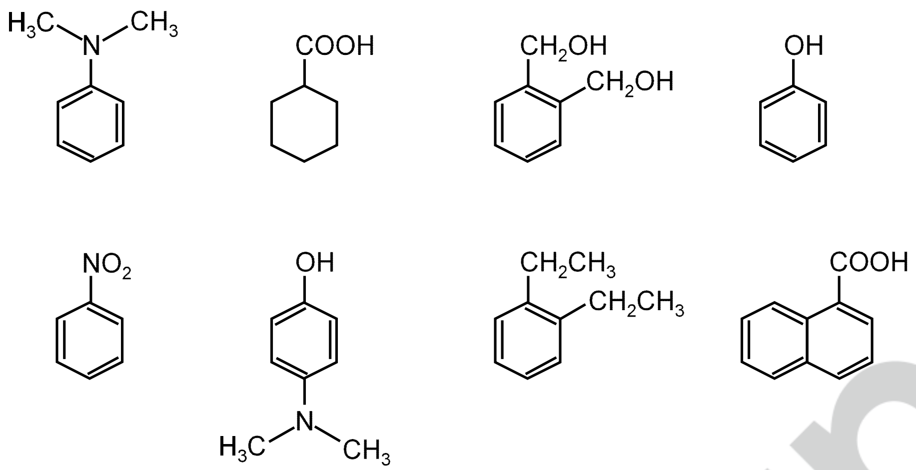
This Section contains **TEN** questions. The answer to each question is a **single digit integer** ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

19. In the scheme given below, the total number of intramolecular aldol condensation products formed from '**Y**' is



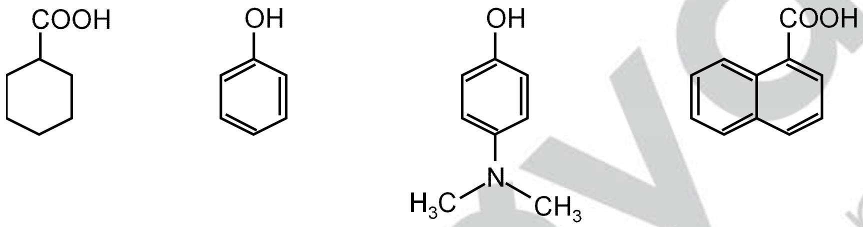
Ans. 1

20. Amongst the following, the total number of compounds soluble in aqueous NaOH is



Ans. **4**

Sol.



21. Amongst the following, the total number of compounds whose aqueous solution turns red litmus paper blue is

KCN	K_2SO_4	$(NH_4)_2C_2O_4$	NaCl	$Zn(NO_3)_2$
$FeCl_3$	K_2CO_3	NH_4NO_3	LiCN	

Ans. **3**

22. Based on VSEPR theory, the number of 90-degree F–Br–F angles in BrF_5 is

Ans. **0**

23. The value of n in the molecular formula $Be_nAl_2Si_6O_{18}$ is

Ans. **3**

24. A student performs a titration with different burettes and finds titre values of 25.2 mL, 25.25 mL, and 25.0 mL. The number of significant figures in the average titre value is

Ans. **3**

25. The concentration of R in the reaction R $\rightarrow$ P was measured as a function of time and the following data is obtained:

[R] (molar)	1.0	0.75	0.40	0.10
t (min.)	0.0	0.05	0.12	0.18

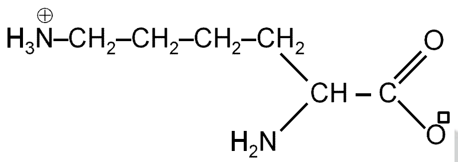
The order of the reaction is:

Ans. **0**

26. The number of neutrons emitted when ${}^{235}_{92}U$ undergoes controlled nuclear fission to ${}^{142}_{54}Xe$ and ${}^{90}_{38}Sr$ is

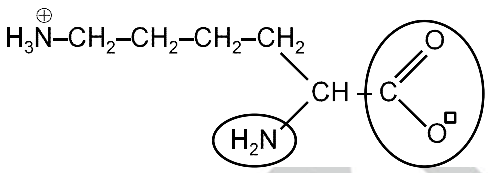
Ans. **4**

27. The total number of basic groups in the following form of lysine is



Ans. **2**

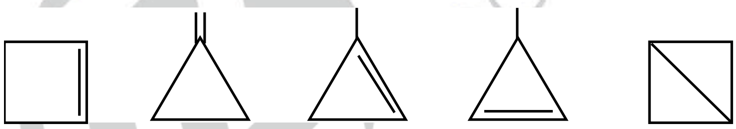
Sol.



28. The total number of cyclic isomers possible for a hydrocarbon with the molecular formula C_4H_6 is

Ans. **5**

Sol.



PART-B: MATHEMATICS

SECTION-I

(**Single Correct Choice Type)**

This section contains **8 multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct. **[3 (–1)]**

29. The value of $\lim\limits_{x\to 0}\dfrac{1}{x^3}\displaystyle\int_0^x \dfrac{t\ln(1+t)}{t^4+4}dt$ is

(A) 0 (B) $\dfrac{1}{12}$ (C) $\dfrac{1}{24}$ (D) $\dfrac{1}{64}$

Sol. **[B]**

$$\lim_{x\to 0}\frac{\displaystyle\int_0^x \frac{t\ln(1+t)}{t^4+4}dt}{x^3} \qquad \left(\frac{0}{0}\right) \text{ form}$$

Applying L'Hospital's Rule once

$$\lim_{x\to 0}\frac{\dfrac{x\ln(1+x)}{(x^4+4)}}{3x^2} = \lim_{x\to 0}\frac{1}{3}\frac{\ln(1+x)}{x}\frac{1}{4+x^4} = \frac{1}{3}\cdot 1\cdot\frac{1}{4+0} = \frac{1}{12}$$

30. The number of 3 × 3 matrices A whose entries are either 0 or 1 and for which the system $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has exactly two distinct solutions, is

(A) 0 (B) $2^9 - 1$ (C) 168 (D) 2

Sol. **[A]**

No such matrix is possible as 3 distinct plane cannot have exactly two solutions.

31. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i}-\hat{j}$, $4\hat{i}$, $3\hat{i}+3\hat{j}$ and $-3\hat{i}+2\hat{j}$ respectively. The quadrilateral PQRS must be a

(A) parallelogram, which is neither a rhombus nor a rectangle.

(B) square.

(C) rectangle, but not a square.

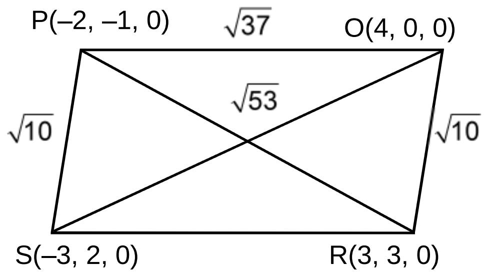
(D) rhombus, but not a square.

Sol. **[A]**

P(–2, – 1); Q(4, 0); R (3, 3); S(–3, 2)

$m_{PQ} = \dfrac{0-(-1)}{4-(-2)} = \dfrac{1}{6}$ $\qquad$ $m_{PR} = \dfrac{3-(-1)}{3-(-2)} = \dfrac{4}{5}$

$m_{RS} = \frac{2-3}{3-3} = \frac{1}{6}$ $\qquad$ $m_{QS} = \frac{2-0}{-3-4} = \frac{-2}{7}$



$m_{PS} = \frac{2-(-1)}{-3-(-2)} = -3$ $\qquad$ $m_{PR} \cdot m_{QS} \neq -1$

$m_{QR} = \frac{0-3}{4-3} = -3$ $\qquad$ $\therefore$ not a rhombus

$\because$ $m_{PQ} \cdot m_{ps} \neq -1$ $\qquad$ $\therefore$ not a rectangle or square

$\therefore$ PQRS is a parallelogram but not a rectangle or rhombus.

32. Let ω be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times. If r_1, r_2 and r_3 are the numbers obtained on the die, then the probability that $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$ is

(A) $\frac{1}{18}$ (B) $\frac{1}{9}$ (C) $\frac{2}{9}$ (D) $\frac{1}{36}$

Sol. **[C]**

$n(S) = 6 \cdot 6 \cdot 6 = 216$

$n(A) = 6 \cdot 4 \cdot 2 = 48$ $\qquad$ 1 2 3 4 5 6

$\therefore$ $P(E) = \frac{48}{216}$ $\qquad$ 1, 4 $\rightarrow$ w ; 2, 5 $\rightarrow w^2$; 3, 6 $\rightarrow$ 1

$= \frac{2}{9}$ **Ans.**

33. Equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is

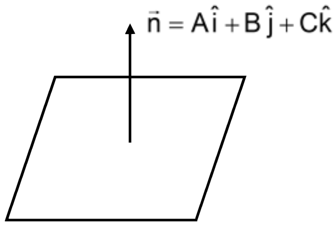
(A) x + 2y – 2z = 0 (B) 3x + 2y – 2z = 0 (C) x – 2y + z = 0 (D) 5x + 2y – 4z = 0

Sol. **[C]**

Plane containing the straight line

$\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ is $Ax + By + Cz = 0$ ....(1)

The vector $\vec{V} = A\hat{i} + B\hat{j} + C\hat{k}$ is perpendicular to $2\hat{i} + 3\hat{j} + 4\hat{k}$



Also (1) is perpendicular to the plane Q containing the lines

$\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$

hence vector normal to Q = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 2 \\ 4 & 2 & 3 \end{vmatrix} = \hat{i}(8) - \hat{j}(1) + \hat{k}(-10) = 8\hat{i} - \hat{j} - 10\hat{k}$

Now $\vec{V}$ is perpendicular to $2\hat{i}+3\hat{j}+4\hat{k}$ as well as $8\hat{i}-\hat{j}-10\hat{k} \Rightarrow \vec{V}$ is collinear with

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 8 & -1 & -10 \end{vmatrix} = -26\hat{i}+52\hat{j}-26\hat{k}$$

$\therefore$ A, B, C are $\propto$ to 1, – 2, 1

Hence required plane is x – 2y + 2 = 0 **Ans.**

34. If the angles A, B and C of a triangle are in an arithmetic progression and if a, b and c denote the lengths of the sides opposite to A, B and C respectively, then the value of the expression $\frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A$, is

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) 1 (D) $\sqrt{3}$

Sol. **[D]**

$E = \frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A = \frac{\sin A}{\sin C}(2\sin C\cos C) + \frac{\sin C}{\sin A}(2\sin A\cos A)$ **[T/S] done**

= 2 sin(A + C) = 2 sin(π – B) = 2 sin B

As A + B + C = π and A + C = 2B $\Rightarrow$ B = 60°

$\therefore$ E = 2 sin 60° = $2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$ **Ans.**

35. Let f, g and h be real-valued functions defined on the interval [0, 1] by f (x) = $e^{x^2} + e^{-x^2}$, g (x) = $xe^{x^2} + e^{-x^2}$ and $h(x) = x^2e^{x^2} + e^{-x^2}$. If a, b and c denote respectively, the absolute maximum of f, g and h on [0, 1], then

(A) a = b and c $\neq$ b (B) a = c and a $\neq$ b (C) a $\neq$ b and c $\neq$ b (D) a = b = c

Sol. **[D]**

f (x) = $e^{x^2} + e^{-x^2}$

f ' (x) = $2xe^{x^2} - 2xe^{-x^2}$

f ' (x) = $2x\left(e^{x^2} - e^{-x^2}\right)$

f ' (x) > 0 $\forall$ x $\in$ [0, 1]

so, f $(x)]_{max}$ = e + $\frac{1}{e}$ = a

g (x) = $xe^{x^2} + e^{-x^2}$

g ' (x) = $2x^2e^{x^2} + e^{x^2} - 2xe^{-x^2}$

g ' (x) > 0 $\forall$ x $\in$ [0, 1]

$g(x)]_{max} = e + \frac{1}{e} = b$

$h(x) = x^2 e^{x^2} + e^{-x^2}$

$h'(x) = 2x^3 e^{x^2} + 2x e^{x^2} - 2x e^{-x^2}$

$h'(x) > 0 \ \forall \ x \in [0, 1]$

$h(x)]_{max} = h(1) = e + \ = c$

so, $a = b = c$ **Ans.**

36. Let p and q be real numbers such that $p \neq 0$, $p^3 \neq q$ and $p^3 \neq -q$. If α and β are nonzero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is

(A) $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$

(B) $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

(C) $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$

(D) $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$

Sol. **[B]**

Sum = $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ or Sum = $\frac{\alpha^2 + \beta^2}{\alpha\beta}$; Given $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$

$\alpha^2 + \beta^2 + 2\alpha\beta = p^2$ ....(1)

$(\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta) = q$

$\alpha^2 + \beta^2 - \alpha\beta = \frac{-q}{p}$ ....(2)

(1) – (2) gives $3\alpha\beta = p^2 + \frac{q}{p} = \frac{p^3 + q}{p} \Rightarrow \alpha\beta = \frac{p^3 + q}{3p}$

required equation is $x^2 - \left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right)x + 1 = 0$ ....(3)

from (2), we get $\frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{-q}{p\alpha\beta} + 1 = \frac{-3pq}{p(p^3 + q)} + 1 = \frac{p^3 - 2q}{p^3 + q}$

equation $x^2 - \frac{(p^3 - 2q)}{p^3 + q}x + 1 = 0$

$(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

SECTION-II

(Multiple Correct Choice Type)

This section contains **5 multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE** may be correct.

[3 (0), Partial marks will be awarded for partially correct answers.]

37. Let ABC be a triangle such that $\angle ACB = \frac{\pi}{6}$ and let a, b and c denote the lengths of the sides opposite to A, B and C respectively. The value(s) of x for which $a = x^2 + x + 1$, $b = x^2 - 1$ and $c = 2x + 1$ is/are

(A) $-\left(2+\sqrt{3}\right)$ (B) $1+\sqrt{3}$ (C) $2+\sqrt{3}$ (D) $4\sqrt{3}$

Sol. **[B]**

$\cos\frac{\pi}{6} = \frac{a^2+b^2-c^2}{2ab} = \frac{\sqrt{3}}{2}$ $\Rightarrow$ $a^2 + b^2 - c^2 = \sqrt{3}\,ab$ or $\left(x^2+x+1\right)^2 + (x^2-1)^2 - (2x+1)^2$

$= \sqrt{3}\left(x^2+x+1\right)(x^2-1)$ or $\left\{(x^2+x+1)^2 - (2x+1)^2\right\} + (x^2-1)^2 = \sqrt{3}(x^2+x+1)(x^2-1)$

or $(x^2 + 3x + 2)(x^2 - x) + (x^2 - 1)^2 = \sqrt{3}(x^2+x+1)(x^2-1)$

or $(x + 2)(x + 1)\, x\, (x - 1) + (x^2 - 1)^2 = \sqrt{3}\ (x^2 + x + 1)(x^2 - 1)$

or $x^2 + 2x + (x^2 - 1) = \sqrt{3}\ (x^2 + x + 1)$

or $2x^2 + 2x - 1 = \sqrt{3}(x^2+x+1)$

or $\left(2-\sqrt{3}\right)x^2 + \left(2-\sqrt{3}\right)x - \left(\sqrt{3}+1\right) = 0$

only positive value of $x = 1+\sqrt{3}$ **Ans.**

Note: Other value of $x = -\left(2+\sqrt{3}\right)$ is rejected (think!)

38. Let f be a real valued function defined on the interval $(0, \infty)$ by $f(x) = \ln x + \int_0^x \sqrt{1+\sin t}\, dt$. Then which of the following statement(s) is/are true?

(A) $f''(x)$ exists for all $x \in (0, \infty)$

(B) $f'(x)$ exists for all $x \in (0, \infty)$ and f' is continuous on $(0, \infty)$ but not differentiable on $(0, \infty)$.

(C) there exists $\alpha > 1$ such that $|f'(x)| < |f(x)|$ for all $x \in (\alpha, \infty)$

(D) there exists $\beta > 0$ such that $|f(x)| + |f'(x)| \le \beta$ for all $x \in (0, \infty)$

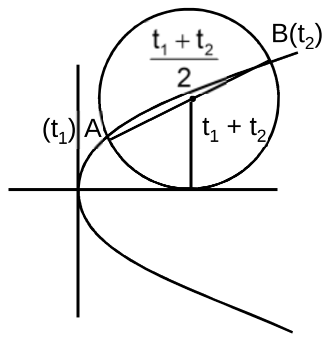
Sol. B, C

39. Let A and B be two distinct point on the parabola $y^2 = 4x$. If the axis of the parabola touches the circle of radius r having AB as its diameter, then the slope of the line joining A and B can be

(A) $\frac{-1}{r}$ (B) $\frac{1}{r}$ (C) $\frac{2}{r}$ (D) $\frac{-2}{r}$

Sol. **[C, D]**

y-coordinates of the centre = radius = r



$\therefore$ $t_1 + t_2 = r$ or -2

$\therefore$ slopes of AB = $\dfrac{2}{t_1+t_2}$

$$= \frac{2}{r} \text{ or } \frac{-2}{r} \quad \Rightarrow \textbf{ C, D}$$

40. The value(s) of $\displaystyle\int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx$ is(are)

(A) $\dfrac{22}{7}-\pi$ (B) $\dfrac{2}{105}$ (C) 0 (D) $\dfrac{71}{15}-\dfrac{3\pi}{2}$

Sol. **[A]**

$$\int_0^1 \frac{x^4(1-2x+x^2)^2}{1+x^2}dx$$

$N^r = x^4\{(1 + x^2)^2 + 4x^2 - 4x(1 + x^2)\}$

$$\int_0^1 \frac{x^4\left\{(1+x^2)^2+4x^2-4x(1+x^2)\right\}}{1+x^2}dx = \int_0^1\left(x^4(1+x^2)+\frac{4x^6}{1+x^2}-4x^5\right)dx$$

$$= \int_0^1\left(x^4+x^6+\frac{4(x^6+1-1)}{1+x^2}-4x^5\right)dx = \int_0^1\left(x^4+x^6+4(x^4-x^2+1)-\frac{4}{1+x^2}-4x^5\right)dx$$

$$= \int_0^1\left(5x^4+x^6-4x^5-4x^2+4-\frac{4}{1+x^2}\right)dx = \left(x^5+\frac{x^7}{7}-\frac{4x^6}{6}-\frac{4x^3}{3}+4x-4\tan^{-1}x\right)_0^1$$

$$1+\frac{1}{7}-\frac{2}{3}-\frac{4}{3}+4-4\cdot\frac{\pi}{4} = 5+\frac{1}{7}-\frac{6}{3}-\pi = \frac{22}{7}-\pi$$

41. Let z_1 and z_2 be two distinct complex numbes and let $z = (1 - t)z_1 + tz_2$ for some real number t with $0 < t < 1$. If Arg(w) denotes the principal argument of a nonzero complex number ω, then

(A) $|z - z_1| + |z - z_2| = |z_1 - z_2|$ (B) Arg $(z - z_1)$ = Arg$(z - z_2)$

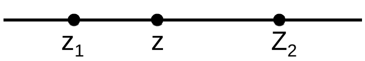
(C) $\begin{vmatrix} z-z_1 & \bar{z}-\bar{z}_1 \\ z_2-z_1 & \bar{z}_2-\bar{z}_1 \end{vmatrix} = 0$ (D) Arg $(z - z_1)$ = Arg$(z_2 - z_1)$

Sol. **[A, C, D]**

$z = (1 - t)z_1 + tz_2$

$z = z_1 + t(z_2 - z_1)$

$z - z_1 = t(z_2 - z_1)$ .....(1)



$\therefore$ $|z-z_1|+|z-z_2|=|z_1-z_2|$ $\therefore$ A is correct

Arg $(z-z_1)$ + arg $(z-z_2) = p$ $\therefore$ B is incorrect

Arg $(z-z_1)$ = Arg(z_2-z_1) $\therefore$ D is correct

from (1) $\bar z-\bar z_1=t(\bar z_2-\bar z_2)$ .....(2)

$$\therefore \quad \frac{z-z_1}{z_2-z_1}=\frac{\bar z-\bar z_1}{\bar z_2-\bar z_1}$$

$$\therefore \quad \begin{vmatrix} z-z_1 & \bar z-\bar z_1 \\ z_2-z_1 & \bar z_2-\bar z_1 \end{vmatrix}=0$$ $\therefore$ C is correct

SECTION-III

Comprehension Type

This section contains 2 groups of questions. Based upon the first paragraph 3 multiple choice questions and based upon the second paragraph 2 multiple questions have to be answered. Each of these question has four choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct. **[3 (–1)]**

Paragraph for questions 42 to 44

Let p be an odd number and T_p be the following set of 2 × 2 matrices.

$$T_p = \left\{A=\begin{bmatrix} a & b \\ c & a \end{bmatrix} : a,b,c \in \{0,1,2,......,p-1\}\right\}$$

42. The number of A in T_p such that A is either symmetric or skew-symmetric or both, and det(A) divisible by p is

(A) $(p-1)^2$ (B) $2(p-1)$ (C) $(p-1)^2+1$ (D) $2p-1$

43. The number of A in T_p such that the trace of A is not divisible by p but det (A) is divisible by p is

[**Note:** The trace of a matrix is the sum of its diagonal entries.]

(A) $(p-1)(p^2-p+1)$ (B) $p^3-(p-1)^2$

(C) $(p-1)^2$ (D) $(p-1)(p^2-2)$

44. The number of A in T_p such that det(A) is not divisible by p is

(A) $2p^2$ (B) p^3-5p (C) p^3-3p (D) p^3-p^2

Sol.

$$A=\begin{bmatrix} a & b \\ c & a \end{bmatrix} : a,b,c \in \{0,1,2,......,p-1$$

$|A| = a^2-bc$

(42) **[D]**

A is either symmetric or skew-symmetric or both, | A | is divisible by P **[T/S] done**

Case-I: A is skew-symmetric or both

If A is skew-symmetric then A will be null matrix which is also a symmetric matrix

Case-II: A is symmetric matrix

$b = c$

(A) is divisible by P

$a^2 - b^2$ is divisible by P

$|A| = (a - b)(a + b)$

$|a - b| \in \{0, 1, 2, \ldots\ldots, p - 1\}$

$|a + b| \in \{0, 1, 2, \ldots\ldots, p, p + 1, \ldots\ldots, 2p - 2\}$

so Case-I $a - b = 0 \Rightarrow$ total = p

Case-II: $a + b = p$

a	b
1	p – 1
2	p – 2
⋮	
p – 1	1

$\Rightarrow$ Total = p – 1

Total 2p – 1 **Ans.**

(43) [C]

Trace = 2a

trace is not divisible by 'P' $\Rightarrow$ $a \neq 0$

so $a = 1, 2, \ldots\ldots, (p - 1)$

for each value of 'a' then all (p – 1) option so that $a^2 - bc$ is divisible by p

e.g. $a = 1$, $p = 7$

$a^2 - bc = 1 - bc$

so $bc = 1 \equiv (1, 1)$

$= 8 \equiv (2, 4), (4, 2)$

$= 15 \equiv (3, 5), (5, 3)$

$= 22 \equiv \phi$

$= 29 \equiv \phi$

$= 36 \equiv (6, 6)$

Total $= 6 = p - 1$

so required $= (p - 1)(p - 1) = (p - 1)^2$ **Ans.**

(44) [D]

Total matrix possible $= p^3$

we exclude all those matrices whose determinant is divisible by p

Case-I: $a = 0$

$a^2 - bc = -bc$

$-bc$ is divisible by p

so $bc = 0$

$b = 0 \Rightarrow c = 0, 1, 2, ..., p-1$

$c = 0 \Rightarrow b = 0, 1, 2, ..., p-1$

Case-II: $a \neq 0$

$\Rightarrow (p-1)^2$ (from Q.43)

Total $= (p-1)^2 + 2p - 1 = p^2$

Required number of matrices whose determinant is not divisible by ' $= p^3 - p^2$ **Ans.**]

Paragraph for questions 45 to 46

The circle $x^2 + y^2 - 8x + 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at the points A and B.

45. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is

(A) $2x - \sqrt{5}\,y - 20 = 0$

(B) $2x - \sqrt{5}\,y + 4 = 0$

(C) $3x - 4y + 8 = 0$

(D) $4x - 3y + 4 = 0$

46. Equation of the circle with AB as its diameter is

(A) $x^2 + y^2 - 12x + 24 = 0$

(B) $x^2 + y^2 + 12x + 24 = 0$

(C) $x^2 + y^2 + 24x - 12 = 0$

(D) $x^2 + y^2 - 24x - 12 = 0$

Sol.

(45) **[B]**

$y = mx + \sqrt{9m^2 - 4}$, centre (4, 0)

$$\left|\frac{4m + \sqrt{9m^2 - 4}}{\sqrt{1 + m^2}}\right| = 4 \quad ....(1)$$

Cross check:

choice (A) satisfies

$$\frac{\frac{8}{\sqrt{5}} + \sqrt{\frac{36}{5} - 4}}{\sqrt{1 + \frac{4}{5}}} = \frac{\frac{8}{\sqrt{5}} + \frac{4}{\sqrt{5}}}{\frac{3}{\sqrt{5}}} = 4$$

(46) **[A]**

$x^2 + y^2 - 8x = 0$ and $\frac{x^2}{9} - \frac{y^2}{4} = 1$

On solving, we get

$13x^2 - 72x - 36 = 0$

On solving x = 6, $\frac{6}{5}$ (y imaginary)

$y = 2\sqrt{3}, -2\sqrt{3}$

$\therefore$ diametric ends $\left(6, 2\sqrt{3}\right)$ and $\left(6, -2\sqrt{3}\right)$

$\therefore$ circle is $(x-6)(x-6) + \left(y-2\sqrt{3}\right)\left(y+2\sqrt{3}\right) = 0$ $\Rightarrow$ $x^2 + y^2 - 12x + 24 = 0$ **Ans.**

SECTION-IV

Integer Type

This section contains **Ten questions**. The answer to each question is a single-digit integer, ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled. **[3 (0)]**

47. If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a} = \frac{\hat{i} - 2\hat{j}}{\sqrt{5}}$ and $\vec{b} = \frac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$, then the value of $\left(2\vec{a} + \vec{b}\right) \cdot \left[\left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} - 2\vec{b}\right)\right]$, is

Sol. $\left(2\vec{a} + \vec{b}\right)\left[\left(\vec{a} \times \vec{b}\right) - 2\left(\vec{a} \times \vec{b}\right) \times \vec{b}\right] = \left(2\vec{a} + \vec{b}\right)\left[(\vec{a} \cdot \vec{a})\vec{b} - \left(\vec{b} \cdot \vec{a}\right)\vec{a} - 2\left(\vec{a} \cdot \vec{b}\right)\vec{b} + 2\left(\vec{b} \cdot \vec{b}\right)\vec{a}\right]$

$= \left(2\vec{a} + \vec{b}\right)\left[\vec{b} - \vec{a} - 2\vec{b} + 2\vec{a}\right] = \left(2\vec{a} + \vec{b}\right) \cdot \left(2\vec{a} + \vec{b}\right) = 4a^2 + b^2 + 0 = 5$ **Ans.**

48. The line 2x + y = 1 is tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If this line passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola, is

Sol. $y = -2x + 1$

$1 = 4a^2 - b^2$

$4a^2 - b^2 = 1$ passes (a/e, 0)

$\frac{2a}{3} = 1 \Rightarrow$ e = 2a

$e^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{4(e^2 - 1)}{e^2}$

$e^2 - 1 = \frac{4(e^2 - 1)}{e^2}$ $\Rightarrow$ e = 2 **Ans.**

49. If the distance between the plane $Ax - 2y + z = d$ and the plane containing the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is $\sqrt{6}$, then | d | is

Sol. $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = \hat{i}(-1) - \hat{j}(-2) + \hat{k}(-1) = \hat{i} - 2\hat{j} + \hat{k} \quad \Rightarrow \quad A = 1$

Plane: $(x - 1) - 2(y - 2) + (z - 3) = 0$

$x - 2y + 2 - 1 + 4 - 3 = 0$

$x - 2y + z = 0$

$\frac{|d - 0|}{\sqrt{6}} = \sqrt{6} \quad \Rightarrow \quad |d| = 6$

50. For any real number x, let [x] denote the largest integer less than or equal to x. Let f be a real valued function defined on the interval [–10, 10] by

$$f(x) = \begin{cases} x - [x] & \text{if } [x] \text{ is odd} \\ 1 + [x] - x & \text{if } [x] \text{ is even} \end{cases}$$

Then the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos \pi x\, dx$ is

Sol. $f(x) = \begin{cases} x - [x] & \text{if } [x] \text{ is odd} \\ 1 + [x] - x & \text{if } [x] \text{ is even} \end{cases}$

If $x \notin I$

$f(-x) = \begin{cases} -x - [-x] & \text{if } [-x] \text{ is odd} \\ 1 + [-x] + x & \text{if } [-x] \text{ is even} \end{cases}$

$= \begin{cases} -x + 1 + [x] & \text{if } [x] \text{ is even} \\ 1 - 1 - [x] + x & \text{if } [x] \text{ is odd} \end{cases}$

$\Rightarrow \quad f(x) = f(-x) \quad$ (even function)

$\Rightarrow \quad 2\frac{\pi^2}{10}\int_0^{10} f(x)\cos \pi x\, dx = \frac{\pi^2}{5}\int_0^{10} \underbrace{f(x)}_{\text{period}=2}\ \underbrace{\cos \pi x}_{\text{period}=1} dx = 5\frac{\pi^2}{5}\int_0^2 f(x)\cos \pi x\, dx$

$= \pi^2\left(\int_0^1 (1 - \{x\})\cos \pi x\, dx + \int_1^2 \{x\}\cos \pi x\, dx\right)$

$= \pi^2\left(\int_0^1 \cos \pi x\, dx - \int_0^1 x\cos \pi x\, dx + \int_1^2 (-1 + x)\cos \pi x\, dx\right)$

$= \pi^2\left(0 - \left\{\frac{x\sin \pi x}{\pi}\Big|_0^1 + \frac{\cos \pi x}{\pi^2}\Big|_0^1\right\} + \left\{0 + \frac{x\sin \pi x}{\pi}\Big|_1^2 + \frac{\cos \pi x}{\pi^2}\Big|_1^2\right\}\right)$

$= \pi^2\left(\frac{2}{\pi^2} + \frac{2}{\pi^2}\right) = 4$ **Ans.**

51. Let w be the complex number $\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$. Then the number of distinct complex number z satisfying

$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}$ = 0 is equal to

Sol. We have $\begin{vmatrix} z+1 & w & w^2 \\ w & z+w^2 & 1 \\ w^2 & 1 & z+w \end{vmatrix} = 0$

Apply $C_1 \longrightarrow C_1 + C_2 + C_3$ and taking out $(z + 1 + w + w^2)$ as common from C_1, we get

$(z + 1 + w + w^2) \begin{vmatrix} 1 & w & w^2 \\ 1 & z+w^2 & 1 \\ 1 & 1 & z+w \end{vmatrix} = 0$

Apply $R_1 \longrightarrow R_1 + R_2 + R_3$, we get

$z\begin{vmatrix} 3 & z & z \\ 1 & z+w^2 & 1 \\ 1 & 1 & z+w \end{vmatrix} = 0$

Expanding along R_1, we get

$z [3 \{(z + w) (z + w^2)–1\}– z \{z + w –1\} + z (1–z–w^2)] = 0$

$\Rightarrow z (z^2) = 0$

$\Rightarrow z^3 = 0$

$\therefore z = 0$

Hence only one value of z will satisfy above equation. **Ans. 1**

52. Let S_k, k = 1, 2, ..., 100, denote the sum of the infinite geometric series whose first term is $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then the value of $\frac{100^2}{100!} + \sum_{k=1}^{100}\left|\left(k^2 - 3k + 1\right)S_k\right|$, is

Sol. $S_k = \frac{(k-1)}{k!}\frac{k}{(k-1)} = \frac{1}{(k-1)!}$

Now $S = \frac{100^2}{100!} + \sum_{K=1}^{100}\frac{|(k^2-3k+1)|}{(k-1)!} = \frac{100^2}{100!} + \sum_{K=1}^{100}\left|\frac{(k-1)(k-2)-1}{(k-1)!}\right| =$

$S = \frac{100^2}{100!} + \sum_{k=3}^{100}\left|\frac{1}{(k-3)!} - \frac{1}{(k-1)!}\right|$

Now $T_1 = \frac{1}{0!} = 1$

$T_2 = \frac{1}{1!} = 1$

$T_3 = \frac{1}{0!} - \frac{1}{2!}$

$T_4 = \frac{1}{1!} - \frac{1}{3!}$

$T_5 = \frac{1}{2!} - \frac{1}{4!}$

$T_6 = \frac{1}{3!} - \frac{1}{5!}$

$\vdots \quad \vdots \quad \vdots$

$T_{100} = \frac{1}{97!} - \frac{1}{99!}$

$\therefore \quad S = \frac{100^2}{100!} + 4 - \frac{(100)^2}{100!} = 2 + 2 - \left(\frac{1}{98!} + \frac{1}{99!}\right) = 4 - \left(\frac{100}{99!}\right) = \left(4 - \frac{(100)^2}{100!}\right) = S = 4$ **Ans.**

53. The number of all possible values of θ, where $0 < \theta < \pi$, for which the system of equations

$(y + z)\cos 3\theta = (xyz) \sin 3\theta$

$x \sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$

$(xyz) \sin 3\theta = (y + 2z) \cos 3\theta + y \sin 3\theta$

have a solution (x_0, y_0, z_0) with $y_0z_0 \neq 0$, is

Sol. $(xyz) \sin 3\theta - y \cos 3\theta - z \cos 3\theta = 0$ ....(i)

$(xyz) \sin 3\theta - (2 \sin 3\theta) y - (2\cos 3\theta) z = 0$ ....(ii)

$(xyz) \sin 3\theta - (\cos 3\theta + \sin 3\theta)y - (2 \cos 3\theta)z = 0$ ....(iii)

$$\begin{vmatrix} \sin 3\theta & \cos 3\theta & \cos 3\theta \\ \sin 3\theta & 2\sin 3\theta & 2\cos 3\theta \\ \sin 3\theta & \cos 3\theta + \sin 3\theta & 2\cos 3\theta \end{vmatrix} = 0$$

$$\Rightarrow \sin 3\theta . \cos 3\theta \begin{vmatrix} 1 & \cos 3\theta & 1 \\ 1 & 2\sin 3\theta & 2 \\ 1 & \cos 3\theta + \sin 3\theta & 2 \end{vmatrix} = 0$$

$$\Rightarrow \sin 3\theta . \cos 3\theta \begin{vmatrix} 1 & \cos 3\theta & -1 \\ 1 & 2\sin 3\theta & 0 \\ 1 & \cos 3\theta + \sin 3\theta & 0 \end{vmatrix} = 0$$

$\Rightarrow \sin 3\theta . \cos 3\theta (\cos 3\theta + \sin 3\theta - 2 \sin 3\theta) = 0$

$\Rightarrow \sin 3\theta . \cos 3\theta (\cos 3\theta - \sin 3\theta) = 0$ ......Equation (A)

From given equations,

if sin 3θ = 0 then equation (2) becomes

$\frac{2\cos 3\theta}{y} = 0 \Rightarrow \text{No solution}$

$\therefore$ sin 3θ ≠ 0

Similarly, if cos 3θ = 0 then equation (1) ⇒ x = 0

for which equation (2) ⇒ 0 = $\frac{2\sin 3\theta}{z}$ which is not possible

$\therefore$ from equation (A)

cos 3θ – sin 3θ = 0

or tan 3θ = 1,

⇒ 3q = $\frac{\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

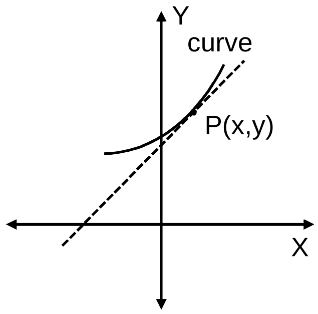
$\therefore$ θ = $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}$

$\therefore$ Number of possible values of θ in (0, π) = 3

54. Let f be a real valued differentiable function on R (the set of all real numbers) such that f (1) = 1. If the y-intercept of the tangent at any point P(x, y) on the curve y = f (x) is equal to the cube of the abscissa of P, then the value of f (–3) is equal to

Sol. (Y – y) = $\frac{dy}{dx}(X - x)$

Put X = 0, Y = y – $\frac{xdy}{dx}$



Now y – $\frac{xdy}{dx} - y = -x^3$

⇒ $\frac{xdy}{dx} - y = -x^3$ ⇒ $\frac{dy}{dx} - \frac{1}{x}y = -x^2$ ..... (i)

$\therefore$ I.F. = $\frac{1}{x}$

Now general solution is

$\frac{y}{x} = -\int x^2 \times \frac{1}{x}dx + C$ ⇒ $\frac{y}{x} = \frac{-x^2}{2} + C$

As f(1) = 1 ⇒ C = $\frac{3}{2}$

$\therefore$ y = $-\frac{1}{2}x^3 + \frac{3}{2}x$

Hence f(–3) = $\frac{27}{2} - \frac{9}{2} = \frac{18}{2} = 9$

55. The number of values of θ in the interval $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for n = 0, ± 1, ± 2 and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$, is

Sol. $\tan\theta = \cot 5\theta \quad \Rightarrow \quad \tan\theta = \tan\left(\frac{\pi}{2} - 5\theta\right)$

$\therefore \quad \theta = n\pi + \frac{\pi}{2} - 5\theta,\ n \in I \quad \Rightarrow \quad \theta = \frac{n\pi}{6} + \frac{\pi}{12},\ n \in I$

$\therefore \quad \theta = \pm\frac{\pi}{12}, \pm\frac{\pi}{4}, \pm\frac{5\pi}{12}$

Now $\quad \sin 2\theta = \cos 4\theta \quad \Rightarrow \quad \cos\left(\frac{\pi}{2} - 2\theta\right) = \cos 4\theta$

$\therefore \quad \frac{\pi}{2} - 2\theta = 2m\pi \pm 4\theta \quad \Rightarrow \quad \frac{\pi}{2} - 2m\pi = 6\theta \text{ or } -2\theta$

$\therefore \quad \theta = \frac{\pi}{12} - \frac{m\pi}{3} \text{ or } \theta = m\pi - \frac{\pi}{12}; m \in I$

$\therefore \quad \theta = \frac{\pi}{12}, -\frac{\pi}{4}, \frac{5\pi}{12}$

$\therefore$ Number of values of θ in $\left(-\frac{\pi}{2}, -\frac{\pi}{2}\right)$ are 3 i.e. $\frac{\pi}{12}, -\frac{\pi}{4}, \frac{5\pi}{12}$. $\Rightarrow$ **Ans. 3**

56. The maximum value of the expression $\frac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$ is

Sol. Maximum value of the given expression will be equal to the reciprocal of the minimum value of

$\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta$

$\therefore \quad E = 1 + \frac{3}{2}\sin 2\theta + 2(1 + \cos 2\theta) = 3 + \frac{3}{2}\sin 2\theta + 2\cos 2\theta = 3 \pm \sqrt{\frac{9}{4} + 4} = 3 \pm \frac{5}{2}$

$\therefore \quad E_{min} = \frac{1}{2}$

Hence $E_{max} = 2$ **Ans.**

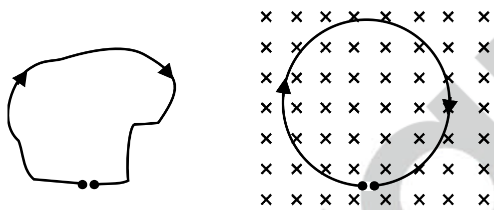
PART-C: PHYSICS

SECTION -I

Single Correct Choice Type

This section contains 8 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

57. A thin flexible wire of length L is connected to two adjacent fixed points and carries a current I in the clockwise direction, as shown in the figure. When the system is put in a uniform magnetic field of strength B going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is



(A) IBL (B) $\frac{IBL}{\pi}$ (C*) $\frac{IBL}{2\pi}$ (D) $\frac{IBL}{4\pi}$

Sol. $2T\cos\left(90^\circ - \frac{d\theta}{2}\right) = I(dL)B$

$\therefore\ 2T\sin\left(\frac{d\theta}{2}\right) = I(Rd\theta)B$

T = IRB 2πR = L

$= \frac{ILB}{2\pi}$

58. An AC voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased

(A) the bulb glows dimmer

(B*) the bulb glows brighter

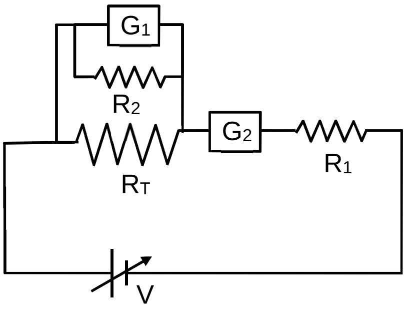
(C) total impedance of the circuit is unchanged

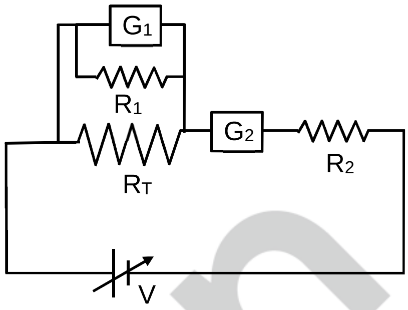
(D) total impedance of the circuit increases

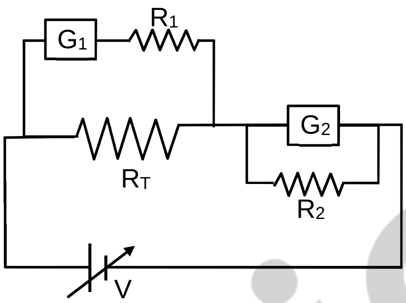
Sol. $z = \frac{V_0}{\sqrt{R^2 + \left(\frac{1}{\omega c}\right)^2}}$ $\omega \uparrow \Rightarrow \frac{1}{\omega c} \downarrow \Rightarrow z \uparrow$

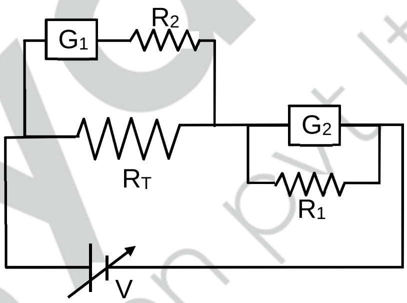
So, i ↑ and hence brightness increases.

59. To verify Ohm's law, a student is provided with a test resistor R_T, a high resistance R_1, a small resistance R_2, two identical galvanometers G_1 and G_2, and a variable voltage source V. The correct circuit to carry out the experiment is :

(A)


(B)


(C*)


(D)


60. Incandescent bulbs are designed by keeping in mind that the resistance of their filament increases with the increase in temperature. If at room temperature, 100 W, 60 W and 40 W bulbs have filament resistances R_{100}, R_{60} and R_{40}, ... respectively, the relation between these resistances is

(A) $\frac{1}{R_{100}} = \frac{1}{R_{40}} + \frac{1}{R_{60}}$

(B) $R_{100} = R_{40} + R_{60}$

(C) $R_{100} > R_{60} > R_{40}$

(D*) $\frac{1}{R_{100}} > \frac{1}{R_{60}} > \frac{1}{R_{40}}$

61. A real gas behaves like an ideal gas if its

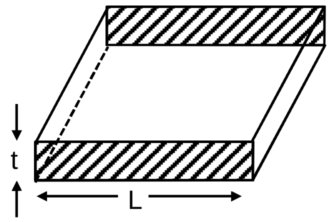
(A) pressure and temperature are both high

(B) pressure and temperature are both low

(C) pressure is high and temperature is low

(D*) pressure is low and temperature is high

62. Consider a thin square sheet of side L and thickness t, made of a material of resistivity ρ. The resistance between two opposite faces, shown by the shaded areas in the figure is

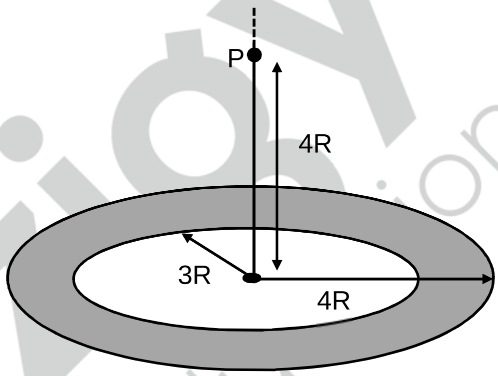


(A) directly proportional to L (B) directly proportional to t

(C*) independent of L (D) independent of t

Sol. $R = \frac{\rho\ell}{A} = \frac{(\rho)L}{Lt} = \frac{\rho}{t}$

63. A thin uniform annular disc (see figure) of mass M has outer radius 4R and inner radius 3R. The work required to take a unit mass from point P on its axis to infinity is

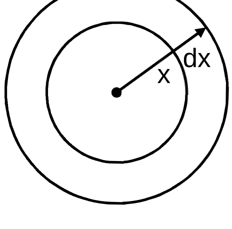


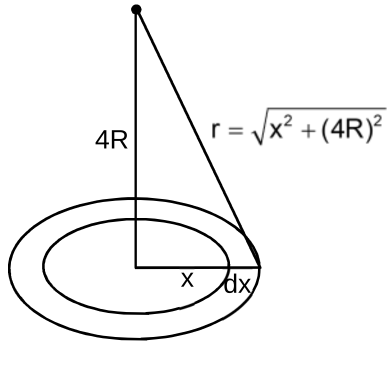
(A*) $\frac{2GM}{7R}\left(4\sqrt{2}-5\right)$ (B) $-\frac{2GM}{7R}\left(4\sqrt{2}-5\right)$ (C) $\frac{GM}{4R}$ (D) $\frac{2GM}{5R}\left(\sqrt{2}-1\right)$

Sol. $W = +\int dv = +\int \frac{Gdm}{r^2}$

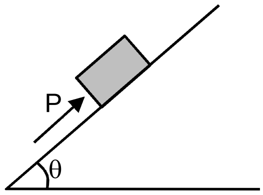
$$= -\int_{3R}^{4R} \frac{G \times \frac{M}{\pi\left(16R^2 - 9R^2\right)} \times 2\pi x dx}{\sqrt{x^2 + 16R^2}}$$

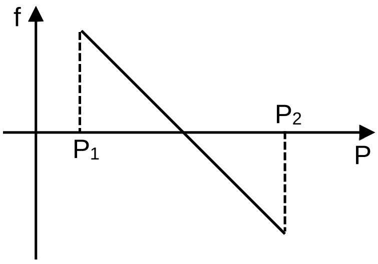
$$= \frac{2GM}{7R^2}[4\sqrt{2}-5]$$

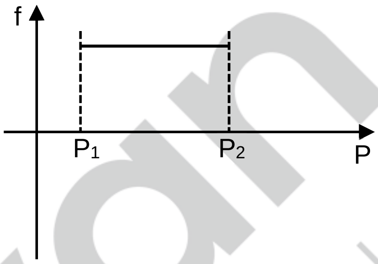


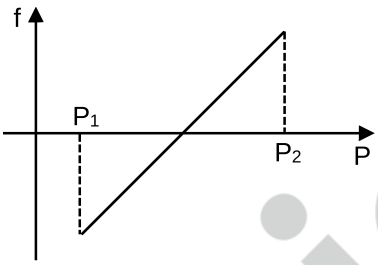


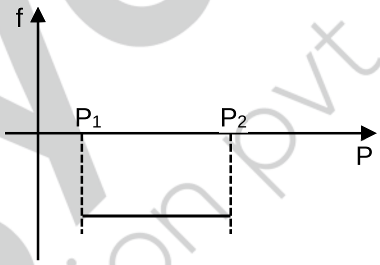
64. A block of mass m is on an inclined plane of angle θ. The coefficient of friction between the block and the plane is μ and $\tan\theta > \mu$. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg(\sin\theta - \mu\cos\theta)$ to $P_2 = mg(\sin\theta + \mu\cos\theta)$, the frictional force f versus P graph will look like



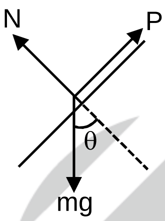
(A*)


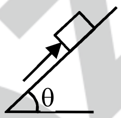
(B)


(C)


(D)


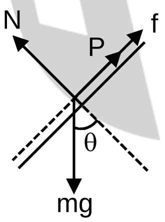
Sol. $\tan\theta \le \mu$

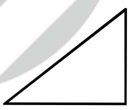




force for which f = 0 $\therefore P_0 = mg\sin\theta$

Case I $P = mg(\sin\theta - \mu\cos\theta) \therefore P < P_0$





$\therefore P + f = mg\sin\theta$

$\Rightarrow mg(\sin\theta - \mu\cos\theta) + f = mg\sin\theta$

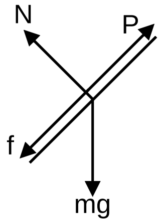
$\therefore f = \mu\, mg\cos\theta$

Case II $P = mg(\sin\theta + \mu\cos\theta) > P_0$

$\therefore P = f + mg\sin\theta$

$mg(\sin\theta + \mu\cos\theta) = f + mg\sin\theta$

$\therefore f = \mu\, mg\cos\theta$



SECTION -II

Multiple Correct Choice Type

This section contains 5 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE OR MORE may be correct.

65. A student uses a simple pendulum of exactly 1m length to determine g, the acceleration due to gravity. He uses a stop watch with the least count of 1 sec for this and records 40 seconds for 20 oscillations. For this observation, which of the following statement(s) is (are) true?

(A*) Error ΔT in measuring T, the time period, is 0.05 seconds

(B) Error ΔT in measuring T, the time period, is 1 second

(C*) Percentage error in the determination of g is 5%

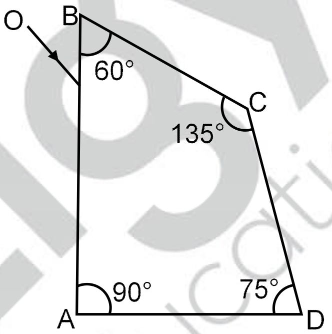
(D) Percentage error in the determination of g is 2.5%

Sol. $T = 2 \pm \frac{1}{20} = 2 \pm 0.05\,\text{sec}$

$$g = \frac{4\pi^2 \ell}{T^2}$$

$$\frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + \frac{2\Delta T}{T} \Rightarrow \frac{\Delta g}{g} = \frac{0.05 \times 2}{2} \qquad \Rightarrow \% = 5\%$$

66. A ray OP of monochromatic light is incident on the face AB of prism ABCD near vertex B at an incident angle of 60° (see figure). If the refractive index of the material of the prism is $\sqrt{3}$, which of the following is (are) correct?



(A*) The ray gets totally internally reflected at face CD

(B*) The ray comes out through face AD

(C*) The angle between the incident ray and the emergent ray is 90°

(D) The angle between the incident ray and the emergent ray is 120°

Sol. 90° + 60° + 75° + x = 360°

x = 360° – 225° = 135°

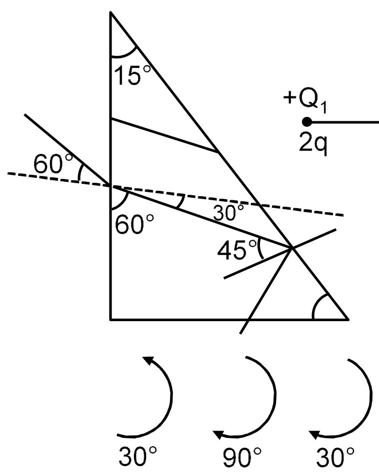
$$\sin\theta_c = \frac{1}{\sqrt{3}} = \frac{1.732}{3} = .58$$

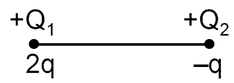
$$\sin 45° = \frac{1}{\sqrt{2}}$$

$$\sin 30° = \frac{1}{2} = .5 \qquad \frac{dx}{\sqrt{x^2 + a^2}}$$

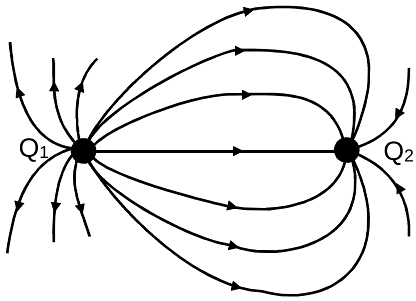
$$\sqrt{3}\sin 30° = 1 \sin\theta$$

$$\therefore \sin\theta = \frac{\sqrt{3}}{2} \qquad \therefore \theta = 60°$$





67. A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the x-axis are shown in the figure. These lines suggest that



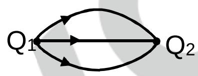
(A*) $|Q_1| > |Q_2|$ (B) $|Q_1| < |Q_2|$

(C) at a finite distance to the left of Q_1 the electric field is zero

(D*) at a finite distance to the right of Q_2 the electric field is zero

Sol. $| Q_1 | > | Q_2 |$

$\therefore$ line density near Q_1 is more



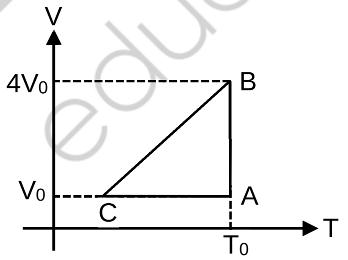
$Q_1 = +ve$

$Q_2 = -\, ve$

and $| Q_1 | > | Q_2 |$

$\therefore$ E = 0 at right of Q_2

68. One mole of an ideal gas in initial state A undergoes 1 a cyclic process ABCA, as shown in the figure. Its pressure at A is P_0 Choose the correct option(s) from the following



(A*) Internal energies at A and B are the same (B*) Work done by the gas in process AB is P_0V_0 ln 4

(C) Pressure at C is $\frac{P_0}{4}$ (D) Temperature at C is $\frac{T_0}{4}$

Sol. T = constant $\therefore$ A

$nR\ell n\left(\frac{uv_0}{v_0}\right)$ $\therefore$ B

69. A point mass of 1kg collides elastically with a stationary point mass of 5kg. After their collision, the 1kg mass reverses its direction and moves with a speed of 2 ms^{-1}. Which of the following statement(s) is (are) correct for the system of these two masses?

(A*) Total momentum of the system is 3kg ms^{-1}

(B) Momentum of 5kg mass after collision is 4kg ms^{-1}

(C*) Kinetic energy of the centre of mass is 0.75 J

(D) Total kinetic energy of the system is 4J

Sol. $v = 5v_2 - 2$ (1)

$v = v_2 + 92$

$2v = 6\,v_2$

$v_2 = v/3$

$v = 5v/3 - 2$

$-2v / 3 = -2$

$v = 3$ m/s

$v_2 = 1$ m/s

$\therefore\ P_i = 3$ $kgms^{-1}$

$k_{cm} = \frac{1}{2}(1+5)v_{cm}^2 = 3v_{cm}^2$

$v_{cm} = \frac{1\times 3}{1+5} = \frac{1}{2} \Rightarrow km = \frac{3}{4} = 0.75$

$\frac{GM_p}{R_p^2} = \frac{\sqrt{6}}{11}\times\frac{GM_e}{R_e^2} \qquad \Rightarrow M_p = \frac{\sqrt{6}}{11}\frac{R_p^2}{R_e^2}M_e$

$\frac{M_p}{R_p^3} = \frac{2}{3}\frac{M_e}{R_e^2} = 1 \qquad M_p = \frac{2}{3}\frac{M_e}{R_e^2}\times R_p^3$

$\frac{2}{3}\frac{M_e}{R_e^3}\times R_p^3 = \frac{\sqrt{6}}{11}\times\frac{R_p^2}{R_e^2}\times M_e$

$R_p = \frac{3\sqrt{6}}{22}R_e$

$\frac{v}{11} = \sqrt{\frac{\sqrt{6}}{11}\times\frac{3\sqrt{6}}{22}} \qquad \Rightarrow v = 11\times\frac{.1}{11}\times\sqrt{\frac{6\times 3}{2}} = 3$

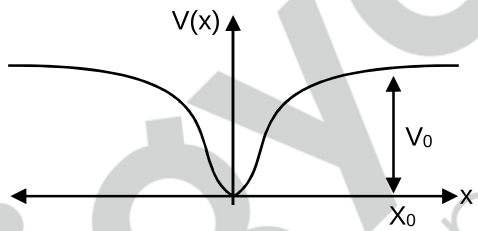
SECTION - III

Paragraph Type

This section contains 2 paragraphs. Based upon the first paragraph 3 multiple choice questions and based upon the second paragraph 2 multiple choice questions have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of which ONLYONE is correct.

Paragraph for Questions 70 to 72

When a particle of mass m moves on the x-axis in a potential of the form $V(x) = kx^2$, it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\frac{m}{k}}$, as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of x = 0 in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x-axis. Its potential energy is $V(x) = \alpha x^4$ ($\alpha > 0$) for | x | near the origin and becomes a constant equal to V_0 for $|x| \geq X_0$ (see figure).



70. If the total energy of the particle is E, it will perform periodic motion only if

(A) $E < 0$ (B) $E > 0$ (C*) $V_0 > E > 0$ (D) $E > V_0$

Sol. $E > 0$ as V(x) is always +ve

$V_0 > E$ as it should not react X_0 as at that point F = 0 on it may go to infinity.

71. For periodic motion of small amplitude A, the time period T of this particle is proportional to

(A) $A\sqrt{\frac{m}{\alpha}}$ (B*) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$ (C) $A\sqrt{\frac{\alpha}{m}}$ (D) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

Sol. $[\alpha] = \frac{[W]}{[L^4]} = \frac{[F]}{[L^3]}$

$[k] = \frac{[F]}{[L]}$

$[\alpha] = \frac{[k]}{[L^2]}$

$[T] = \left[\left(\frac{m}{k}\right)^{1/2}\right] = \left[\left(\frac{m}{\alpha L^2}\right)^{1/2}\right] = \frac{1}{A}\sqrt{\frac{m}{\alpha}}$

72. The acceleration of this particle for | x | > X_0 is

(A) proportional to V_0

(B) proportional to $\frac{V_0}{mX_0}$

(C) proportional to $\sqrt{\frac{V_0}{mX_0}}$

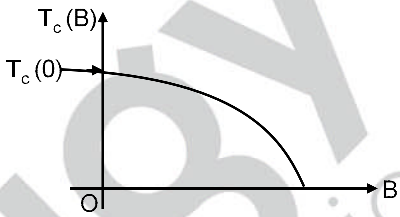
(D*) zero

Sol. U = constant

F = 0

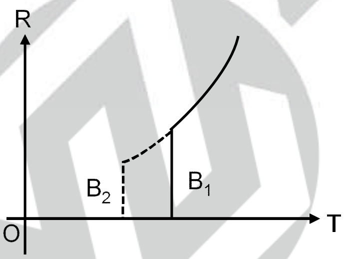
Paragraph for Questions 73 to 74

Electrical resistance of certain materials, known as superconductors, changes abruptly from a nonzero value to zero as their temperature is lowered below a critical temperature $T_C(0)$. An interesting property of superconductors is that their critical temperature becomes smaller than $T_C(0)$ if they are placed in a magnetic field, i.e., the critical temperature $T_C(B)$ is a function of the magnetic field strength B. The dependence of $T_C(B)$ on B is shown in the figure.

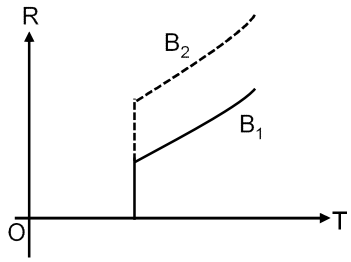


73. In the graphs below, the resistance R of a superconductor is shown as a function of its temperature T for two different magnetic fields B_1 (solid line) and B_2 (dashed line). If B_2 is larger than B_1, which of the following graphs shows the correct variation of R with T in these fields ?

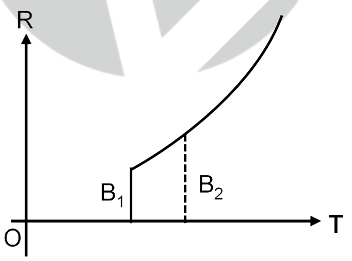
(A*)



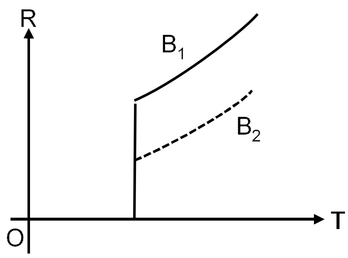
(B)



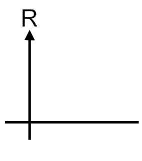
(C)



(D)



Sol. $B_2 > B_1$



$T_{C_2} < T_{C_1}$

74. A superconductor has $T_C(0)$ = 100 K. When a magnetic field of 7.5 Tesla is applied, its T_C decreases to 75K. For this material one can definitely say that when

(A) B = 5 Tesla, T_C (B) = 80 K
(B*) B = 5 Tesla, 75K < T_C (B) < 100 K
(C) B = 10 Tesla, 75 K < T_C (B) < 100 K
(D) B = 10 Tesla, T_C (B) = 70 K

Sol. $(T_C)_5 > (T_C)_{7.5}$

$(T_C)_5 < (T_C)_0$

$\Rightarrow (T_C)_{7.5} < (T_C)_5 < (T_C)_0$

$75k < (T_C)_5 < 100k$

SECTION -IV

Integer Type

This section contains TEN questions. The answer to each question is a single-digit integer, ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

75. When two identical batteries of internal resistance 1Ω each are connected in series across a resistor R, the rate of heat produced in R is J_1. When the same batteries are connected in parallel across R, the rate is J_2. If J_1 = 2.25 J_2 then the value of R in Ω is.

Ans. 4

Sol. $\frac{J_2}{J_1} = \left(\frac{I_2}{I_1}\right)^2 = \frac{1}{2.25} \qquad \Rightarrow \frac{I_2}{I_1} = \frac{1}{1.5}$

$I_1 = \frac{2E}{R+2}$

$I_2 = \frac{E}{R+\frac{1}{2}} = \frac{2E}{2R+1}$

$\Rightarrow \frac{I_2}{I_1} = \frac{R+2}{(2R+1)} \Rightarrow \frac{32}{23} = \frac{R+2}{2R+1}$

14R + 2 = 3R + 6

R = 4

76. Two spherical bodies A(radius 6 cm) and B(radius 18 cm) are at temperature T_1 and T_2, respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B?

Ans. **9**

Sol. $u = 6AT^4$

$$\frac{u_A}{u_B} = \frac{A_A T_A^4}{A_B T_B^4}$$

$$\frac{T_A}{T_B} = \frac{\lambda_B}{\lambda_A} = 3$$

$$\frac{u_A}{u_B} = \frac{A_A}{A_B}(81) = \left(\frac{r_A}{r_B}\right)^2 (81)$$

$$\left(\frac{r_A}{r_B}\right) = \frac{1}{3}$$

$$\frac{u_A}{u_B} = 9$$

77. When two progressive waves y_1 = 4 sin (2x – 6t) and y_2 = 3 sin $\left(2x - 6t - \frac{\pi}{2}\right)$ are superimposed, the amplitude of the resultant wave is

Ans. **5**

Sol. $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\theta}$

$A_1 = 4$

$A_2 = 3$ Þ $A = 5$

$\phi = -\pi/2$

78. A 0.1 kg mass is suspended from a wire of negligible mass. The length of the wire is 1m and its cross-sectional area is 4.9×10^{-7} m^2. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 rad s^{-1}. If the Young's modulus of the material of the wire is $n \times 10^9$ Nm^{-2}, the value of n is

Ans. **4**

Sol. $k = \frac{YA}{L}$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{YA}{mL}}$$

$$140 = \sqrt{\frac{n \times 10^9 \times 4.9 \times 10^{-7}}{0.1 \times 1}} = \sqrt{49n \times 10^2}$$

$140 = 70\sqrt{n}$

$n = 4$

79. A binary star consists of two stars A (mass 2.2 M_S) and B (mass 11 M_S), where M_S is the mass of the sun. They are separated by distance d and are rotating about their centre of mass, which is stationary. The ratio of the total angular momentum of the binary star to the angular momentum of a star B about the centre of mass is

Ans. **6**

Sol. $\frac{m_1 v_1^2}{r_1} = \frac{Gm_1 m_2}{d^2}$

$\frac{v_1^2}{r_1} = \frac{Gm_2}{d^2}$

$v_1 = \sqrt{\frac{Gm_2 r}{d^2}} = \sqrt{\frac{Gm_2}{d}\left(\frac{5}{6}\right)}$

80. Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}$ g, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is $\frac{2}{3}$ times that of the earth. If the escape speed on the surface of the earth is taken to be 11 kms^{-1}, the escape speed on the surface of the planet in kms^{-1} will be

Ans. **3**

Sol. $\frac{V_{C2}}{V_{e\rho}} = \frac{\rho_2 R_2}{\rho_1 R_1}$

$= \sqrt{\frac{\rho_2 R_2}{\rho_1 R_1}}$

$\frac{g_2}{g_1} = \left(\frac{M_2}{M_1}\right)\left(\frac{R_1}{R_2}\right)^2$

$\frac{\rho_2}{\rho_1} = \left(\frac{M_2}{M_1}\right)\left(\frac{R_1}{R_2}\right)^3$

$\frac{g_2 / g_1}{\rho_2 / \rho_1} = \left(\frac{R2}{R_1}\right)$

$\frac{V_{C_2}}{V_{C_1}} = \sqrt{\left(\frac{g_2}{g_1}\right)\left(\frac{g_2}{g_1}\right)\Big/\frac{\rho_2}{\rho_1}}$

$= \frac{g_2}{g_1}\sqrt{\frac{\rho_2}{\rho_1}} = \frac{\sqrt{6}}{11}$

$\sqrt{\frac{3}{2}}$

81. A piece of ice (heat capacity = 2100 J kg^{-1} $°C^{-1}$ and latent heat = 3.36 × 10^5 J kg^{-1}) of mass m grams is at –5°C at atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally when the ice-water mixture is in equilibrium, it is found that 1gm of ice has melted. Assuming there is no other heat exchange in the process, the value of m is

Ans. **8**

Sol. m × 2100 × 5 + 10^{-3} × 3.36 × 10^5 = 420

$$m = \frac{84}{10.5} = 8\text{gm} \ \textbf{Ans.}$$

82. A stationary source is emitting sound at a fixed frequency f_0, which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars is 1.2% of f_0. What is the difference in the speeds of the cars (in km per hour) to the nearest integer? The cars are moving at constant speeds much smaller than the speed of sound which is 330 ms^{-1}.

Ans. **7**

Sol.
$$\rho_1 = \frac{c+v_1}{c-v_1}\rho_0$$

$$\rho_2 = \frac{c+v_2}{c-v_2}\rho_0$$

$$\frac{\Delta\rho}{\rho_0} = \left(\frac{c+v_1}{c-v_1}\right) - \left(\frac{c+v_2}{c-v_2}\right) = \frac{1.2}{100}$$

$$= \frac{c^2 - v_1v_2 + cv_1 - cv_2 - (c^2 - v_1v_2 - cv_1 + cv_2)}{(c-v_1)(c-v_2)}$$

$$= \frac{2c(v_1 - v_2)}{c^2} = \frac{1.2}{100}$$

$$= v_1 - v_2 = \frac{1.2}{100} \times \frac{330}{2} \times \frac{18}{5}$$

$$= \frac{1.2 \times 33 \times 18}{100} \approx 7 \text{ km/hr.} \ \textbf{Ans.}$$

83. The focal length of a thin biconvex lens is 20 cm. When an object is moved from a distance of 25 cm in front of it to 50 cm, the magnification of its image changes from m_{25} to m_{50}. The ratio $\frac{m_{25}}{m_{50}}$ is

Ans. **6**

Sol.
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{u} = \frac{1}{f} + \frac{1}{u}$$

$$\frac{u}{v} = \left(\frac{f+u}{f}\right)$$

$$m = \left(\frac{u}{v}\right) = \left(\frac{f}{f+u}\right)$$

$$m_{25} = \frac{20}{20-25} = -4$$

$$m_{50} = \frac{20}{20-50} = \frac{-2}{3}$$

$$\frac{m_{25}}{m_{50}} = \frac{-4}{-2/3} = 6$$ **Ans.**

84. An α-particle and a proton are accelerated from rest by a potential difference of 100V. After this, their de-Broglie wavelengths are λ_α and λ_p respectively. The ratio $\frac{\lambda_p}{\lambda_\alpha}$, to the nearest integer, is

Ans. **3**

Sol. $KE = \frac{1}{2}mv^2 = \frac{1}{2m}p^2$

$p = \sqrt{2mKE}$

$2e \to \alpha \to 4m$

$e\ p \to m$

KE = 2 e × 100

Ke = e × 100

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mKE}}$$

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{2\times 4m\times 2\times e\times 100}{2\times m\times e\times 100}} = 2\times 1.414 = 2.8 \to 3$$ **Ans.**