JEE MAINS-2013

ONLINE_23-04-2013

IMPORTANT INSTRUCTIONS

1. Immediately fill in the particulars on this page of the Test Booklet with **Blue/Black Ball Point Pen. Use of pencil is strictly prohibited.**
2. The test is of **3** hours duration.
3. The Test Booklet consists of **90** questions. The maximum marks are **360**.
4. There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for each correct response.
5. Candidates will be awarded marks as stated above in instruction No.5 for correct response of each question. 1/4 (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
6. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 5 above.

PART-A-PHYSICS

1. A particle of mass 2 kg is moving such that at time t, its position, in meter, is given by $\vec{r}(t) = 5\hat{i} - 2t^2\hat{j}$. The angular momentum of the particle at t = 2s about the origin in kg m^{-2} s :

(1) $(10\hat{i} - 16\hat{j})$ (2) $-40\hat{k}$ (3) $40\hat{k}$ (4) $-80\hat{k}$

2. Consider a finite insulated, uncharged conductor placed near a finite positively charged conductor. The uncharged body must have a potential :

(A) more than the charged conductor and more than at infinity.

(B) less than the charged conductor and less than at infinity.

(C) more than the charged conductor and less than at infinity.

(D*) less than the charged conductor and more than at infinity.

3. Wax is coated on the inner wall of a capillary tube and the tube is then dipped in water. Then, compared to the unwaxed capillary, the angle of contact θ and the height h upto which water rises change. These changes are :

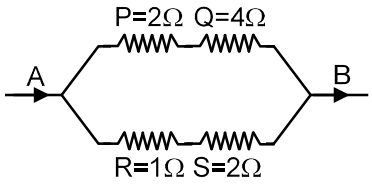
(1) θ decreases and h also decreases (2) θ decreases and h increases

(3) θ increases and h also increases (4*) θ increases and h decreases

4. A body of mass 'm' is tied to one end of a spring and whirled round in a horizontal plane with a constant angular velocity. The elongation in the spring is 1 cm. If the angular velocity is doubled, the elongation in the spring is 5 cm. The original length of the spring is :

(1) 12 cm (2) 15 cm (3) 10 cm (4) 16 cm

5. Which of the four resistances P, Q, R and S generate the greatest amount of heat when a current flows A to B ?



(1) P (2) Q (3) R (4*) S

6. Electrons are accelerated through a potential difference V and protons are accelerated through a potential difference 4 V. The de-Broglie wavelengths are λ_e and λ_p for electrons and protons respectively. The ratio of $\frac{\lambda_e}{\lambda_p}$ is given by : (given m_e is mass of electron and m_p is mass of proton).

(1) $\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}}$ (2) $\frac{\lambda_e}{\lambda_p} = \frac{1}{2}\sqrt{\frac{m_e}{m_p}}$ (3) $\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_e}{m_p}}$ (4) $\frac{\lambda_e}{\lambda_p} = 2\sqrt{\frac{m_p}{m_e}}$

7. A liquid drop having 6 excess electrons is kept stationary under a uniform electric field of 25.5 kV m^{-1}. The density of liquid is 1.26×10^3 kg m^{-3}. The radius of the drop is (neglect buoyancy) :

(1) 7.8×10^{-7} m (2) 4.3×10^{-7} m (3) 0.078×10^{-7} m (4) 3.4×10^{-7} m

8. A 70 kg man leaps vertically into the air from a crouching position. To take the leap the man pushes the ground with a constant force F to raise himself. The centre of gravity rises by 0.5 m before he leaps. After the leap the c.g. rises by another 1 m. The maximum power delivered by the muscles is :

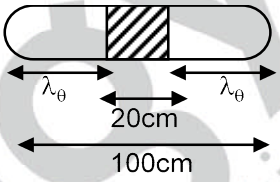
(Take g = 10 ms^{-2}) :

(1) 6.26×10^3 Watts at take off (2) 6.26×10^4 Watts at take off

(3) 6.26×10^3 Watts at the start (4) 6.26×10^4 Watts at the start

9. A thin tube sealed at both ends is 100 cm long. It lies horizontally, the middle 20 cm containing mercury and two equal ends containing air at standard atmospheric pressure. If the tube is now turned to a vertical position, by what amount will the mercury be displaced ?

(Given : cross section of the tube can be assumed to be uniform)



(1) 0.0 cm (2) 2.95 cm (3) 8.65 cm (4) 5.18 cm

10. 500 g of water and 100 g of ice at 0°C are in a calorimeter whose water equivalent is 40 g. 10 g of steam at 100°C is added to it. Then water in the calorimeter is :

(Latent heat of ice = 80 cal/g, Latent heat of steam = 540 cal/g) ?

(1) 610 g (2) 600 g (3) 580 g (4) 590 g

11. A sonometer wire of length 114 cm is fixed at both the ends. Where should the two bridges be placed so as to divide the wire into three segments whose fundamental frequencies are in the ratio 1 : 3 : 4 ?

(1) At 72 cm and 96 cm from one end (2) At 48 cm and 96 cm from one end

(3) At 24 cm and 72 cm from one end (4) At 36 cm and 84 from one end

12. If the time period t of the oscillation of a drop of liquid of density d, radius r, vibrating under surface tension s is given by the formula $t = \sqrt{r^{2b}\, s^{c}\, d^{a/2}}$. It is observed that the time period is directly proportional to $\sqrt{\frac{d}{s}}$.

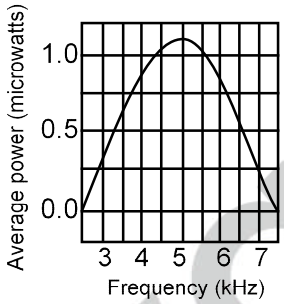
The value of b should therefore be:

(A*) $\frac{3}{2}$ (B) $\frac{3}{4}$ (C) $\sqrt{3}$ (D) $\frac{2}{3}$

13. A projectile of mass M is fired so that the horizontal range is 4 km. At the highest point the projectile explodes in two parts of masses M/4 and 3M/4 respectively and the heavier part starts falling down vertically with zero initial speed. The horizontal range (distance from point of firing) of the lighter part is :

(1) 10 km (2) 16 km (3) 1 km (4) 2 km

14. The plot given below is of the average power delivered to an LRC circuit versus frequency. The quality factor of the circuit is :

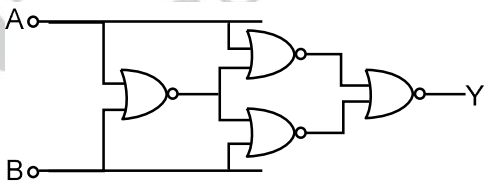


(1*) 2.0 (2) 0.4 (3) 2.5 (4) 5.0

15. In the Bohr's model of hydrogen-like atom the force between the nucleus and the electron is modified as $F = \frac{e^2}{4\pi\varepsilon_0}\left(\frac{1}{r^2}+\frac{\beta}{r^3}\right)$, where β is a constant. For this atom, the radius of the n^{th} orbit in terms of the Bohr radius $\left(a_0 = \frac{\varepsilon_0 h^2}{m\pi e^2}\right)$:

(A) $r_n = a_0\, n - \beta$ (B) $r_n = a_0\, n^2 + \beta$ (C*) $r_n = a_0\, n^2 - \beta$ (D) $r_n = a_0\, n^2 + \beta$

16. A system of four gates is set up as shown. The 'truth table' corresponding to this system is :



(1)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

(2*)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(3)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(4)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

17. A parallel plate capacitor of area 60 cm^2 and separation 3 mm is charged initially to 90 µC. If the medium between the plate gets slightly conducting and the plate loses the charge initially at the rate of 2.5×10^{-8}C/s, then what is the magnetic field between the plates ?

(1) 1.63×10^{-11} T (2*) Zero (3) 2.0×10^{-7} T (4) 2.5×10^{-8}T

18. n identical waves each of intensity I_0 interfere with each other. The ratio of maximum intensities if the interference is (i) coherent and (ii) incoherent is :

(1) n^2 (2) $\frac{1}{n}$ (3*) n (4) $\frac{1}{n^2}$

19. Bob of a simple pendulum of length ℓ is made of iron. The pendulum is oscillating over a horizontal coil carrying direct current. If the time period of the pendulum is T then :

(1) $T < 2\pi\sqrt{\frac{\ell}{g}}$ and damping is smaller than in air alone

(2) $T > 2\pi\sqrt{\frac{\ell}{g}}$ and damping is smaller than in air alone

(3*) $T < 2\pi\sqrt{\frac{\ell}{g}}$ and damping is larger than in air alone

(4) $T = 2\pi\sqrt{\frac{\ell}{g}}$ and damping is larger than in air alone

20. This question has statement-1 and statement-2. Of the four choice given after the statements, choose the one that best describes the two statements.

Statement1 : The internal energy of a perfect gas is entirely kinetic and depends only on absolute temperature of the gas at constant pressure is lower than that at constant volume:

Statement2 : A perfect gas is heated keeping pressure constant and later a constant volume. For the same amount of heat the temperature of the gas at constant pressure is lower than that at constant volume.

(1) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation of statement-1

(2) Statement-1 is false, statement-2 is true

(3) Statement-1 is false, statement-2 true but statement-2 is not the correct explanation of statement-1

(4) Statement-1 is true, statement-2 is false

21. A copper wire of length 1.0 m and a steel wire of length 0.5 m having equal cross-sectional area are joined end to end. The composite wire is stretched by a certain load which stretches the copper wire by 1 mm. If the young's modulii of copper and steel are respectively 1.0×10^{11} Nm^{-2} and 2.0×10^{11} Nm^{-2}, the total extension of the composite wire is :

(1) 1.75 mm (2) 1.25 (3) 2.0 mm (4) 1.50 mm

22. Select the correct statement from the following:

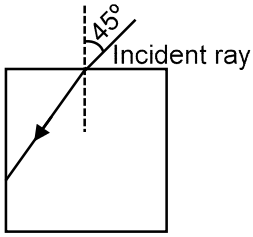
(1) electromagnetic waves are longitudinal waves

(2) Electromagnetic waves cannot travel in vacuum

(3) Electromagnetic waves are produced by charges moving with uniform velocity

(4*) Electromagnetic waves carry both energy and momentum as they propagate through space.

23. A light ray falls on a square glass slab as shown in the diagram. The index of refraction of the glass, if total internal reflection is to occur at the vertical face, is equal to :



(1) $\frac{3}{2}$ (2) $\sqrt{\frac{3}{2}}$ (3) $(\sqrt{2}+1)/2$ (4) $\sqrt{\frac{5}{2}}$

24. Which of the following statement is NOT correct ?

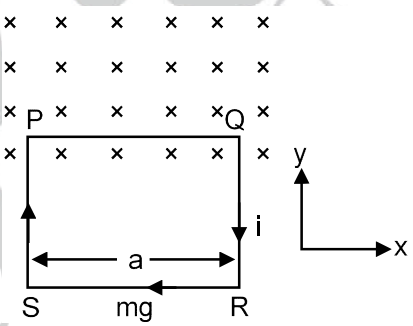
(1) Skywave signals in the broadcast frequency range are stronger at night than in the day time

(2*) The critical frequency of an ionosphere layer is the highest frequency that will be reflected by the layer when it is vertically incident

(3) electromagnetic waves of frequencies higher than about 30 MHz cannot penetrate the ionosphere

(4) Ground wave signals are more stable than the sky wave signals.

25. A rectangular loop of wire, supporting a mass m, hangs with one end in a uniform magnetic field $\vec{B}$ pointing out of the plane of the paper. A clockwise current is set up such that i > mg/Ba, where a is the width of the loop. Then :



(1) The magnetic causes force with both a vertical and horizontal component along PQ, Since the displacement is vertical no work is current

(2) The magnetic field causes force with booth a vertical and horizontal component. Work has to be done by an external agent to maintain current

(3) The weight rises due to a vertical force caused by the magnetic field but no work is done on the system

(4) The weight rises due to a vertical force caused by the magnetic field and work is extracted from the magnetic field

26. This question has statement-1 and statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement 1 : Very large size telescopes are reflecting telescopes instead of refracting telescopes.

Statement 2 : It is easier to provide mechanical support to large size mirrors than large size lenses.

(1) Statement-1 and Statement-2 are true and Statement-2 is not the correct explanation for statement-1

(2*) Statements-1 and statement-2 are true and Statement-2 is correct explanation for Statement-1

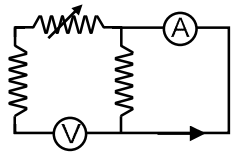
(3) Statement-1 is false and Statement-2 is true

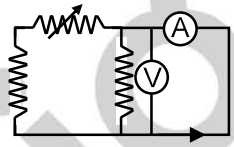
(4) Statement-1 is ture and Statement-2 is false

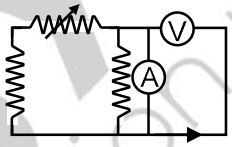
27. A particle of charge 16×10^{-16} C moving with velocity 10 ms^{-1} along x-axis enters a region where magnetic field of induction $\vec{B}$ is along the y-axis and an electric field of magnitude 10^4 Vm^{-1} is along the negative z-axis. If the charged particle continues moving along x-axis, the magnitude of $\vec{B}$ is :

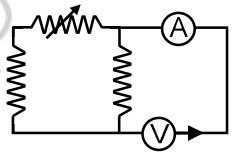
(1) 2×10^3 Wb m^{-2} (2) 16×10^3 Wb m^{-2} (3) 1×10^3 Wb m^{-2} (4) 4×10^3 Wb m^{-2}

28. Correct set up to verify Ohm's law is :

(1) 

(2) 

(3) 

(4) 

29. A body of mass 20 kg is standing on a 80 kg free to move long cart. There is negligible friction between cart and ground. Initially, the boy is standing 25 m from a wall. If he walks 10 m on the cart towards the wall, then the final distance of the boy from the wall will be :

(1) 15.5 m (2) 12.5 m (3) 17 m (4) 15 m

30. The ratio of the coefficient of volume expansion of a gas container to that of a viscous liquid kept inside the container is 1 : 4. What fraction of the inner volume of the container should the liquid occupy so that the volume of the remaining vacant space will be same at all temperatures ?

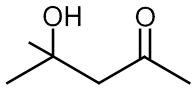
(1) 2 : 5 (2) 1 : 8 (3) 1 : 4 (4) 1 ; 64

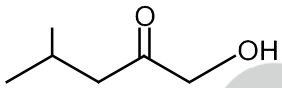
PART-B-CHEMISTRY

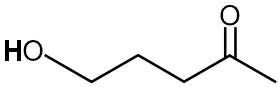
31. Bond distance in HF is 9.17×10^{-11} m. Dipole moment of HF is 6.104×10^{-30} Cm. The percent ionic character in HF will be : (electron charge = 1.60×10^{-19} C)

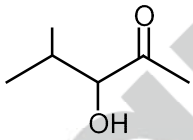
(1) 61.0 % (2) 38.0% (3) 35.5 % (4) 41.5 %

32. Which of the following is the product of aldol condensation ?

(1) 

(2) 

(3) 

(4) 

33. The shape of IF_6^- is :

(1) Pyramidal (2) Octahedral

(3) Trigonally distorted octahedron (4) Square antiprism

34. The de Broglie wavelength of a car of mass 1000 kg and velocity 36 km/hr is :

($h = 6.63 \times 10^{-34}$ Js)

(1) 6.626×10^{-30} m (2) 6.626×10^{-31} m (3) 6.626×10^{-38} m (4) 6.626×10^{-34} m

35. Monocarboxylic acids are functional isomers of :

(1) Esters (2) Amines (3) Ethers (4) Alcohols

36. Number of atoms in the following samples of substances is the largest in :

(1) 48.0 g of magnesium (2) 127.0 g of iodine

(3) 4.0 g of hydrogen (4) 71.0 g of chlorine

37. The correct order of viscosity of the following liquids will be :

(1) dimethyl ether < methyl alcohol < water < glycerol

(2) methyl alcohol < glycerol < water < dimethyl ether

(3) glycerol < dimethyl ether < water < methyl alcohol

(4) Water < methyl alcohol < dimethyl ether < glycerol

38. Which of the following polymer is a polyamide?

(1) Terylene (2) Rubber

(3) Nylon (4) Vulcanised rubber

39. What is the pH of a 10^{-4} M OH^- solution at 330K, if K_w at 330 K is $10^{-13.6}$?

(1) 9.0 (2) 10 (3) 9.6 (4) 4

40. The numbers of protons, electrons and neutrons in a molecule of heavy water are respectively :

(1) 8, 10, 11 (2) 11, 10, 10 (3) 10, 11, 10 (4) 10, 10, 10

41. Which has trigonal bipyramidal shape ?

(1) $XeOF_2$ (2) XeO_3F_2 (3) XeO_3 (4) $XeOF_4$

42. Natural glucose is termed D-glucoe because :

(1) –OH on the second carbon is on the right side in Fischer projection

(2) It is dextrorotatory

(3) –OH on the fifth carbon is on the right side in Fishcger projection

(4) –OH on the sixth carbon is on the right side in Fischer projection

43. Smoke is an example of :

(1) Solid dispersed in solid (2) Solid dispersed in gas

(3) Gas dispersed in liquid (4) Gas dispersed in solid

44. When a small amount of $KMnO_4$ is added to concentrated H_2SO_4, a green oily compound is obtained which is highly explosive in nature. Compound may be :

(1) MnO_2 (2) Mn_2O_7 (3) $MnSO_4$ (4) Mn_2O_3

45. H_1–Receptor antagonists is a term associated with :

(1) Antihistamins (2) Analgesics (3) Antacids (4) Antiseptics

46. The order of reactivity of the given haloalkanes towards nucleophile is :

(1) RBr > FI > RCl (2) RI > RBr > RCl (3) RBr > RCl > RI (4) RCl > RBr > RI

47. Glycosidic linkage is a actually an :

(1) Carbonyl bond (2) Amide bond (3) Ester bond (4) Ether bond

48. In a face centered cubic lattice, atoms of A form the corner points and atoms of B form the face centered points. If two atoms of A are missing from the corner points, the formula of the ionic compounds is :

(1) AB_2 (2) AB_4 (3) A_2B_5 (4) AB_3

49. The reaction of phenol with benzoyl chloride to give phenyl benzoate is known as :

(1) Claisen reaction (2) Gatterman-Koch reaction

(3) Schotten-Baumann reaction (4) Reimer-Tiemann reaction

50. $N_2(g) + 3H_2(g) \rightleftharpoons 2\,NH_3(g)$, K_1 (1)

$N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$, K_2 (2)

$H_2(g) + \frac{1}{2}O_2(g) \rightleftharpoons H_2O(g)$, K_3 (3)

The equation for the equilibrium constant of the reaction $2\,NH_3(g) + \frac{5}{2}\,O_2(g) \rightleftharpoons 2NO(g) + 3H_2O(g)$, (K_4) in terms of K_1, K_2 and K_3 is:

(1) $K_1\,K_2\,K_3$ (2) $\frac{K_1K_3^2}{K_2}$ (3) $\frac{K_2K_3^3}{K_1}$ (4) $\frac{K_1K_2}{K_3}$

51. Identify incorrect statement :

(1) Copper (I) compound are diamagnetic

(2) Copper(I) compound are colouless except where colour results from change transfer

(3) Cu_2O is colourless

(4) Cu_2S is black

52. Equimolar solutions of the following compounds are prepared separately in water. Which will have the lowest pH value ?

(1) $CaCl_2$ (2) $BeCl_2$ (3) $MgCl_2$ (4) $SrCl_2$

53. Given :

$$E^0_{\frac{1}{2}Cl_2/Cl^-} = 1.36V, E^0_{Cr^{3+}/Cr} = 0.74V\ ,\ E^0_{Cr_2O_7^{2-}/Cl^{3+}} = 1.33V, E^0_{MnO_4^-/Mn^{2+}} = 1.51V$$

The correct order of reducing power of the species (Cr, Cr^{3+}, Mn^{2+} and Cl^-) will be:

(1) $Cr^{3+} < Cl^- < Cr < Mn^{2+}$ (2) $Mn^{2+} < Cl^- < Cr^{3+} < Cr$

(3) $Mn^{2+} < Cr^{3+} < Cl^- < Cr$ (4) $Cr^{3+} < Cl^- < Mn^{2+} < Cr$

54. The order of basicity of amines in gaseous state is :

(1) $NH_3 > 1^o > 2^o > 3^o$ (2) $3^o > 2^o > NH_3 > 1^o$

(3) $1^o > 2^o > 3^o > NH_3$ (4) $3^o > 2^\circ > 1^o > NH_3$

55. Which of the following statements/relationships is not correct in thermodynamic changes ?

(1) $w = -nRT\ln\frac{V_2}{V_1}$ (Isothermal reversible expansion of an ideal gas.)

(2) For a system at constant volume, heat involved merely changes of internal energy.

(3) $\Delta U = 0$ (Isothermal reversible expansion of a gas.)

(4) $q = -nRT\ln\frac{V_2}{V_1}$ (Isothermal reversible expansion of an ideal gas.)

56. Identify the incorrect statement :

(1) $(Si_3O_9)^{6-}$ has cyclic structure

(2) Trialkylchlorosilance on hydrolysis gives R_3SiOH

(3) In $(Si_3O_9)^{6-}$, tetrahedral SiO_4 units share two oxygen atoms

(4) $SiCl_4$ undergoes hydrolysis to give H_4SiO_4

57. Which is the correct order of second ionization potential of C, N, O and F in the following ?

(1) C > N > O > F (2) F > O > N > C (3) O > N > F > C (4) O > F > N > C

58. The rate constant of a zero order reaction is 2.0×10^{-2} mol L^{-1} s^{-1}. If the concentration of the reactant after 25 seconds is 0.5 M. What is the initial concentration ?

(1) 12.5 M (2) 0.5 M (3) 1.0 M (4) 1.25 M

59. Vapour pressure of pure benzene is 119 torr and that of toluene is 37.0 torr at the same temperature. Mole fraction of toluene in vapour phase which is in equilibrium with a solution of benzene and toluene having a mole fraction of toluene 0.50, will be :

(1) 0.137 (2) 0.205 (3) 0.435 (4) 0.237

60. The magnetic moment of the complex anion $[Cr(NO)(NH_3)(CN)_4]^{2-}$ is :

(1) 1.73 BM (2) 3.87 BM (3) 2.82 BM (4) 5.91 BM

PART-C-MATHEMATICS

61. If $a_1, a_2, a_3, \ldots\ldots, a_n, \ldots\ldots$ are in A.P. such that $a_4 - a_7 + a_{10} = m$, then the sum of first 13 terms of this A.P. is :

(1*) 13 m (2) 12 m (3) 15 m (4) 10 m

62. On the sides AB, BC, CA of a ΔABC, 3, 4, 5 distinct points (excluding vertices A, B, C) are respectively chosen. The number of triangles that can be constructed using these chosen points as vertices are :

(1) 215 (2*) 205 (3) 210 (4) 220

63. The point of intersection of the normals to the parabola $y^2 = 4x$ at the ends of its latus rectum is :

(1) (2, 0) (2*) (3, 0) (3), (0, 3) (4) (0, 2)

64. Let $a = \text{Im}.\left(\frac{1+z^2}{2iz}\right)$, where z is any non-zero complex number. The set A = { a : | z | = 1 and $z \neq \pm 1$} is equal to :

(1*) (– 1, 1) (2) [0, 1) (3) [– 1, 1] (4) (– 1, 0]

65. If two lines L_1 and L_2 in space, are defined by

$L_1 = \left\{ x = \sqrt{\lambda}\, y + (\sqrt{\lambda} - 1), z = (\sqrt{\lambda} - 1)\, y + \sqrt{\lambda} \right\}$ and

$L_2 = \left\{ x = \sqrt{\mu}\, y + (1 - \sqrt{\mu}), z = (1 - \sqrt{\mu})\, y + \sqrt{\mu} \right\}$

then L_1 is perpendicular to L_2, for all non-negative reals λ and μ, such that :

(1*) $\lambda = \mu$ (2) $\lambda + \mu = 0$ (3) $\lambda \neq \mu$ (4) $\sqrt{\lambda} + \sqrt{\mu} = 1$

66. If $\vec{a}$ and $\vec{b}$ are non-collinear vectors, then the value of α for which the vectors $\vec{\mu} = (\alpha - 2)\vec{a} + \vec{b}$ and $\vec{v} = (2 + 3\alpha)\vec{a} - 3\vec{b}$ are collinear is :

(1) $\frac{3}{2}$ (2*) $\frac{2}{3}$ (3) $\frac{-2}{3}$ (4) $\frac{-3}{2}$

67. If the circle $x^2 + y^2 - 6x - 8y + (25 - a^2) = 0$ touches the axis of x, then a equals :

(1) ± 3 (2*) ± 4 (3) ± 2 (4) 0

68. The sum of the series ; $(2)^2 + 2(4)^2 + 3(6)^2 + \ldots\ldots$ upto 10 terms is :

(1) 11300 (2*) 12100 (3) 12300 (4) 11200

69. **Statement-1:** The system of linear equations

$x + (\sin\alpha)\, y + (\cos\alpha)\, z = 0$

$x + (\cos\alpha)\, y + (\sin\alpha)\, z = 0$

$x - (\sin\alpha)\, y - (\cos\alpha)\, z = 0$

has a non-trivial solution for only one value of α lying in the interval $\left(0, \frac{\pi}{2}\right)$.

Statement-2: The equation in $\alpha \begin{vmatrix} \cos\alpha & \sin\alpha & \cos\alpha \\ \sin\alpha & \cos\alpha & \sin\alpha \\ \cos\alpha & -\sin\alpha & -\cos\alpha \end{vmatrix} = 0$ has only one solution lying in the interval $\left(0, \frac{\pi}{2}\right)$.

(1) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for statement-1

(2) Statement-1 is true; Statement-2 is false.

(3*) Statement-1 is false; Statement-2 is true.

(4) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1

70. The least integral value α of x such that $\frac{x-5}{x^2+5x-14} > 0$, satisfies :

(1) $\alpha^2 + 3\alpha - 4 = 0$ (2) $\alpha^2 - 7\alpha + 6 = 0$ (3) $\alpha^2 - 5\alpha + 4 = 0$ (4*) $\alpha^2 + 5\alpha - 6 = 0$

71. The value of $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx$ is :

(1) 4π (2) $\frac{\pi}{2}$ (3) π (4*) $\frac{\pi}{4}$

72. Let f be a composite function of x defined by $f(u) = \frac{1}{u^2+u-2}, u(x) = \frac{1}{x-1}$. Then the number of points x where f is discontinuous is :

(1) 2 (2) 1 (3*) 3 (4) 4

73. If the median and the range of four numbers {x, y, 2x +y, x – y}, where $0 < y < x < 2y$ are 10 and 28 respectively, then the mean of the numbers is :

(1) 5 (2*) 14 (3) 10 (4) 18

74. A, B, C try to hit a target simultaneously but independently. Their respective probabilities of hitting the targets are $\frac{3}{4}, \frac{1}{2}, \frac{5}{8}$. The probability that the target is hit by A or B but not by C is :

(1) $\frac{9}{64}$ (2) $\frac{7}{8}$ (3*) $\frac{21}{64}$ (4) $\frac{7}{32}$

75. If $S = \tan^{-1}\left(\frac{1}{n^2+n+1}\right) + \tan^{-1}\left(\frac{1}{n^2+3n+3}\right) + \ldots + \tan^{-1}\left(\frac{1}{1+(n+19)(n+20)}\right)$, then tan S is equal to :

(1*) $\frac{20}{n^2+20n+1}$ (2) $\frac{n}{401+20n}$ (3) $\frac{n}{n^2+20n+1}$ (4) $\frac{20}{401+20n}$

76. The number of solutions of the equation $\sin 2x - 2\cos x + 4\sin x = 4$ in the interval $[0, 5\pi]$ is :

(1) 4 (2) 6 (3) 5 (4*) 3

77. The integral $\int \frac{x}{2-x^2+\sqrt{2-x^2}}\,dx$ equals :

(1) $-x\log\left|1-\sqrt{2-x^2}\right| + C$ (2) $\log\left|1+\sqrt{2+x^2}\right| + C$

(3*) $-\log\left|1+\sqrt{2-x^2}\right| + C$ (4) $x\log\left|1-\sqrt{2+x^2}\right| + C$

78. If the projections of a line segment on the x, y and z-axes in 3-dimensional space are 2, 3 and 6 respectively, then the length of the line segment is :

(1) 6 (2) 12 (3) 9 (4*) 7

79. If the curves $\frac{x^2}{\alpha} + \frac{y^2}{4} = 1$ and $y^3 = 16x$ intersect at right angles, then a value of α is :

(1) $\frac{3}{4}$ (2) 2 (3*) $\frac{4}{3}$ (4) $\frac{1}{2}$

80. For integers m and n, both greater than 1, consider the following three statements.

P : m divides n

Q : m divides n^2

R : m is prime,

then

(1) $Q \rightarrow R$ (2) $Q \rightarrow P$ (3) $P \wedge Q \rightarrow R$ (4*) $Q \wedge R \rightarrow P$

81. Let A, other than I or – I, be a 2 × 2 real matrix such that $A^2 = I$, I being the unit matrix. Let Tr(A) be the sum of diagonal elements of A.

Statement-1: Tr(A) = 0

Statement-2: det.(A) = – 1

(1) Statement-1 is true; Statement-2 is false.

(2) Statement-1 is false; Statement-2 is true.

(3*) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for statement-1

(4) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1

82. The area under the curve $y = |\cos x - \sin x|$, $0 \le x \le \frac{\pi}{2}$ and above x-axis is :

(1) $2\sqrt{2}$ (2*) $2\sqrt{2}-2$ (3) 0 (4) $2\sqrt{2}+2$

83. Let θ_1 be the angle between two lines $2x + 3y + c_1 = 0$ and $-x + 5y + c_2 = 0$ and θ_2 be the angle between two lines $2x + 3y + c_1 = 0$ and $-x + 5y + c_3 = 0$, where c_1, c_2, c_3 are any real numbers.

Statement-1: If c_2 and c_3 are proportional, then $\theta_1 = \theta_2$.

Statement-2: $\theta_1 = \theta_2$ for all c_2 and c_3.

(1*) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1

(2) Statement-1 is false; Statement-2 is true.

(3) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for statement-1

(4) Statement-1 is true; Statement-2 is false.

84. The cost of running a bus from A to B, is Rs. $\left(av+\frac{b}{v}\right)$, where v km/h is the average speed of the bus. When the bus travels at 30 km/h, the cost comes out to be Rs. 75 while at 40 km/h, it is Rs. 65. Then the most economical speed (in km/h) of the bus is :

(1*) 60 (2) 40 (3) 45 (4) 50

85. If $f(x) = \sin(\sin x)$ and $f''(x) + \tan x\, f'(x) + g(x) = 0$, then $g(x)$ is :

(1) $\cos^2 x \cos(\sin x)$ (2) $\sin^2 x \sin(\cos x)$ (3) $\sin^2 x \cos(\cos x)$ (4*) $\cos^2 x \sin(\sin x)$

86. The sum of the rational terms in the binomial expansion of $\left(2^{\frac{1}{2}}+3^{\frac{1}{5}}\right)^{10}$ is :

(1) 32 (2) 9 (3*) 41 (4) 25

87. Let $R = \{(x, y) : x, y \in N \text{ and } x^2 - 4xy + 3y^2 = 0\}$, where N is the set of all natural numbers. Then the relation R is :

(1) reflexive and transitive

(2) symmetric and transitive

(3*) reflexive but neither symmetric nor transitive

(4) reflexive and symmetric

88. A tangent to the hyperbola $\frac{x^2}{4}-\frac{y^2}{2}=1$ meets x-axis at P and y-axis at Q. Lines PR and QR are drawn such that OPRQ is a rectangle (where O is the origin). Then R lies on :

(1*) $\frac{4}{x^2}-\frac{2}{y^2}=1$ (2) $\frac{2}{x^2}-\frac{4}{y^2}=1$ (3) $\frac{4}{x^2}+\frac{2}{y^2}=1$ (4) $\frac{2}{x^2}+\frac{4}{y^2}=1$

89. If the extremities of the base of an isosceles triangle are the points (2a, 0) and (0, a) and the equation of one of the sides is x = 2a, then the area of the triangle, in square units, is :

(1) $\frac{25a^2}{4}$ (2) $\frac{5}{4}a^2$ (3*) $\frac{5}{2}a^2$ (4) $5a^2$

90. If a curve passes through the point $\left(2, \frac{7}{2}\right)$ and has slope $\left(1-\frac{1}{x^2}\right)$ at any point (x, y) on it, then the ordinate of the point on the curve whose abscissa is – 2 is :

(1*) $\frac{-3}{2}$ (2) $\frac{5}{2}$ (3) $\frac{3}{2}$ (4) $\frac{-5}{2}$