JEE MAIN-2017

IMPORTANT INSTRUCTIONS

1. Immediately fill in the particulars on this page of the Test Booklet with **Blue/Black Ball Point Pen. Use of pencil is strictly prohibited.**
2. The test is of **3** hours duration.
3. The Test Booklet consists of **90** questions. The maximum marks are **360**.
4. There are **three** parts in the question paper A, B, C consisting of **Mathematics, Physics and Chemistry** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for each correct response.
5. Candidates will be awarded marks as stated above in instruction No.5 for correct response of each question. 1/4 (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
6. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 5 above.

PART-A-MATHEMATICS

1. If S is the set of distinct values of 'b' for which the following system of linear equations

x + y + z = 1

x + ay + z = 1

ax + by + z = 0

has no solution, then S is

(1) an empty set

(2) an infinite set

(3) a finite set containing two or more elements

(4) a singleton

Ans. [4]

Sol. $\because$ Equations has no solutions D = 0 and at least one of D_x, D_y, $D_z \neq 0$

$$\therefore \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix} = 0$$

$\Rightarrow 1(a - b) - 1(1 - a) + 1(b - a^2) = 0$

$\Rightarrow 2a - b - 1 + b - a^2 = 0 \Rightarrow a^2 - 2a + 1 = 0 \Rightarrow a = 1$

$\because$ Two planes coincide for no solution third plane should be parallel to first two plane

$$\therefore \frac{1}{a} = \frac{1}{b} = \frac{1}{1} \Rightarrow b = 1$$

$\therefore$ A singleton set.

2. The following statement $(p \to q) \to [(\sim p \to q) \to q]$ is

(1) a tautology

(2) equivalent to $\sim p \to q$

(3) equivalent to $p \to \sim q$

(4) a fallacy

Ans. [1]

Sol. $(p \to q) \to [(\sim p \to q) \to q]$

p	q	~ p	$\sim p \to q$	$(\sim p \to q) \to q$	$p \to q$	$(p \to q)[\sim p \to q) \to q$
T	T	F	T	T	T	T
T	F	F	T	F	F	T
F	T	T	T	T	T	T
F	F	T	F	T	T	T

$\therefore$ Given statement is a tautology.

3. If 5 $(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of cos 4x is

(1) $\frac{-3}{5}$ (2) $\frac{1}{3}$ (3) $\frac{2}{9}$ (4) $\frac{-7}{9}$

Ans. [4]

Sol. $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$

$$\Rightarrow 5\left(\frac{1-\cos 2x}{1+\cos 2x} - \frac{1+\cos 2x}{2}\right) = 2\cos 2x + 9$$

$$\Rightarrow 5\left(\frac{2-2\cos 2x-(1+2\cos 2x+\cos^2 2x)}{2\,(1+\cos 2x)}\right) = 2\cos 2x + 9$$

$\Rightarrow 5(1 - 4\cos 2x - \cos^2 2x) = 2(2\cos^2 2x + 11\cos 2x + 9)$

$\Rightarrow 9\cos^2 2x + 42\cos 2x + 13 = 0$

$\Rightarrow 9\cos^2 2x + 3\cos 2x + 39\cos 2x + 13 = 0$

$\Rightarrow (3\cos 2x + 1)(3\cos 2x + 13) = 0$

$\Rightarrow \cos 2x = \frac{-1}{3}$ $\quad (\because -1 \le \cos 2x \le 1)$

$\therefore \cos 4x = 2\cos^2 2x - 1 = \frac{2}{9} - 1 = \frac{-7}{9}.$

4. For three events A, B and C,

P (Exactly one of A or B occurs) = P (Exactly one of B or C occurs)

= P(Exactly one of C or A occurs) = $\frac{1}{4}$ and P (All the three events occur simultaneously) = $\frac{1}{16}$.

Then the probability that at least one of the events occurs, is

(1) $\frac{7}{32}$ (2) $\frac{7}{16}$ (3) $\frac{7}{64}$ (4) $\frac{3}{16}$

Ans. [2]

Sol. P (exactly one of A & B occur) = $P(A) + P(B) - 2P(A \cap B) = \frac{1}{4}$ …….(1)

P (exactly one of B & C occur) = $P(B) + P(C) - 2P(B \cap C) = \frac{1}{4}$ …….(2)

P (exactly one of C & A occur) = $P(C) + P(A) - 2P(C \cap A) = \frac{1}{4}$ …….(3)

Adding (1), (2) & (3)

$$2(P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A)) = \frac{3}{4}$$

$\Rightarrow P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) = \frac{3}{8}$

P(All three occurs simultaneously) = $P(A \cap B \cap C) = \frac{1}{16}$

$\therefore$ P (atleast one of events occur) = $P(A \cup B \cup C) = \frac{3}{8} + \frac{1}{16} = \frac{7}{16}$. **Ans.**

5. Let ω be a complex number such that $2\omega + 1 = z$ where $z = \sqrt{-3}$. If $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix}$ = 3k, then k is equal to

(1) – z (2) z (3) – 1 (4) 1

Ans. [1]

Sol. $\because$ $2\omega + 1 = \sqrt{3}\, i \Rightarrow \omega = \frac{-1+\sqrt{3}i}{2}$ = imaginary cube root of unity

$$\therefore \begin{vmatrix} 1 & 1 & 1 \\ 1 & -(1+\omega)^2 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix} = 3k$$

applying $R_1 \rightarrow R_1 + R_2 + R_3$

$$\therefore k = \omega^2 - \omega = \frac{-1-\sqrt{3}i}{2} - \left(\frac{-1+\sqrt{3}i}{2}\right) = -\sqrt{3}i = -z.$$. **Ans.**

6. Let k be an integer such that triangle with vertices (k, – 3k), (5, k) and (– k, 2) has area 28 sq. units. Then the orthocentre of this triangle is at the point

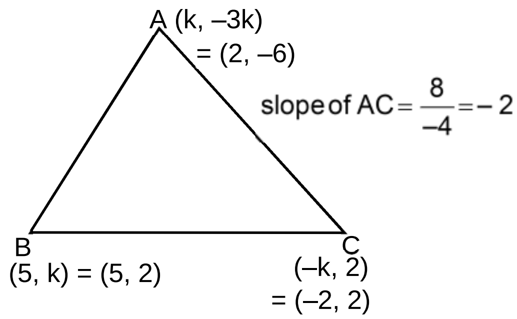
(1) $\left(2, \frac{-1}{2}\right)$ (2) $\left(1, \frac{3}{4}\right)$ (3) $\left(1, \frac{-3}{4}\right)$ (4) $\left(2, \frac{1}{2}\right)$

Ans. [4]

Sol. Area $= \frac{1}{2}\begin{vmatrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{vmatrix} = 28$

$\Rightarrow |\, k\,(k - 2) + 3k\,(5 + k) + 1\,(10 + k^2)\,| = 56$

$\Rightarrow 5k^2 + 13k + 10 = \pm 56$



(i) $5k^2 + 13k + 66 = 0$

$\because$ D < 0

$\therefore$ No real root

(ii) $5k^2 + 13k - 46 = 0$

$5k^2 + 23k - 10k - 46 = 0$

$\Rightarrow (5k + 23)(k - 2) = 0 \Rightarrow k = 2$ ($\because k \in I$)

$\therefore$ points are = (2, – 6), (5, 2) and (– 2, 2)

$\therefore$ equation of altitude through A $\Rightarrow$ x = 2

and equation of altitude through B $\Rightarrow y - 2 = +\left(\frac{1}{2}\right)(x - 5)$

for point of orthocentre x = 2 and $y - 2 = \frac{1}{2}(-3) \Rightarrow y = \frac{1}{2}$

$\therefore$ Orthocentre = $\left(2, \frac{1}{2}\right)$. **Ans.**

7. Twenty meters of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower-bed, is

(1) 12.5 (2) 10 (3) 25 (4) 30

Ans. [3]

Sol. $\because 2r + r\theta = 20$

$$\text{Area} = \frac{1}{2}r^2\theta = \frac{1}{2}r^2\left(\frac{20 - 2r}{r}\right)$$

$$A = \frac{1}{2} r(20 - 2r) = 10r - r^2$$

$$\therefore \frac{dA}{dr} = 10 - 2r$$

$\therefore \frac{dA}{dr} = 0$ at r = 5 and having a maximum as $\frac{d^2A}{dr^2}$ is negative.

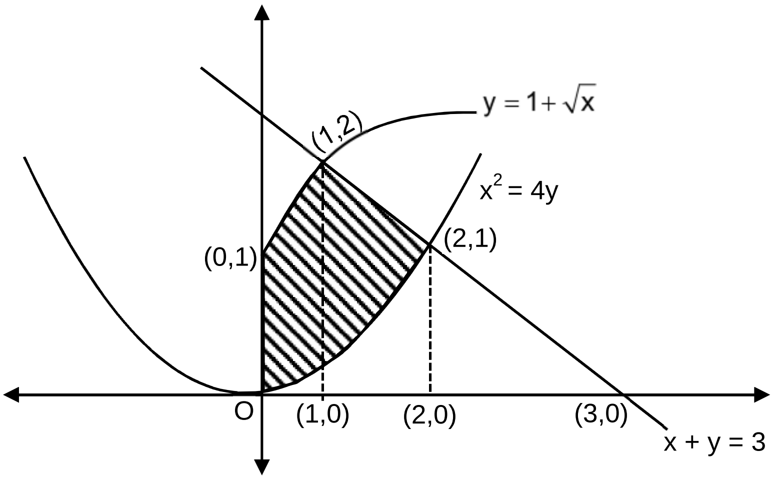
$\therefore A_{max} = 10 \times 5 - 5^2 = 25$. **Ans.**

8. The area (in sq. units) of the region $\{(x, y) : x \geq 0, x + y \leq 3, x^2 \leq 4y \text{ and } y \leq 1 + \sqrt{x}\}$ is

(1) $\frac{59}{12}$ (2) $\frac{3}{2}$ (3) $\frac{7}{3}$ (4) $\frac{5}{2}$

Ans. [4]

Sol. Required area $\int_0^1 (1 + \sqrt{x})\,dx + \int_1^2 (3 - x)\,dx - \int_0^2 \frac{x^2}{4}\,dx$



$$=\left(x+\frac{2x^{\frac{3}{2}}}{3}\right)_0^1+\left(3x-\frac{x^2}{2}\right)_1^2-\left(\frac{x^3}{12}\right)_0^2=\frac{5}{3}+\left(4-\left(3-\frac{1}{2}\right)\right)-\left(\frac{8}{12}-0\right)$$

$$=\frac{5}{3}+4-\frac{5}{2}-\frac{2}{3}=\frac{5}{2}$$. **Ans.**

9. If the image of the point P (1, – 2, 3) in the plane, 2x + 3y – 4z + 22 = 0 measured parallel to the line, $\frac{x}{1}=\frac{y}{4}=\frac{z}{5}$ is Q, then PQ is equal to

(1) $3\sqrt{5}$ (2) $2\sqrt{42}$ (3) $\sqrt{42}$ (4) $6\sqrt{5}$

Ans. [2]

Sol. Equation of line parallel to given line and passing through P(1, – 2, 3) will be

$$\frac{x-1}{1}=\frac{y+2}{4}=\frac{z-3}{5}=\lambda$$

Let its point of intersection with plane 2x + 3y – 4z + 22 = 0 be M (λ + 1, 4λ – 2, 5λ + 3)

$\because$ It satisfies the plane

$\therefore$ 2λ + 2 + 12λ – 6 – 20λ – 12 + 22 = 0

$\Rightarrow$ – 6λ + 6 = 0 $\Rightarrow$ λ = 1

$\therefore$ Point m = (2, 2, 8)

Let Q (α, β, γ) be image.

$\therefore$ mid-point of PQ will be point M.

$$\therefore \frac{\alpha+1}{2}=2\Rightarrow \alpha=3$$

$\frac{\beta-2}{2}=2\Rightarrow\beta=6$ and $\frac{\gamma+3}{2}=8\Rightarrow\gamma=13.$

$\therefore$ Point Q $=\sqrt{2^2+8^2+10^2}=\sqrt{168}=2\sqrt{42}$. Ans.

10. If for $x \in \left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x} \cdot g(x)$, then g(x) equals

(1) $\frac{9}{1+9x^3}$ (2) $\frac{3x\sqrt{x}}{1-9x^3}$ (3) $\frac{3x}{1-9x^3}$ (4) $\frac{3}{1+9x^3}$

Ans. [1]

Sol. Given function = f(x) = $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right), x \in \left(0, \frac{1}{4}\right)$

$$= 2\tan^{-1}(3x\sqrt{x}) \;,\; x \in \left(0, \frac{1}{4}\right)$$

$$\therefore \text{ derivative} = \frac{2}{1+9x^3} \times 3.\frac{3x^{\frac{1}{2}}}{2} = \frac{9}{1+9x^3} \times 3 \cdot \frac{3x^{\frac{1}{2}}}{2} = \frac{9\sqrt{x}}{1+9x^3} = \sqrt{x}\; g(x)$$

$\therefore\; g(x) \;=\; \frac{9}{1+9x^3}$. **Ans.**

11. If (2 + sin x) + (y + 1) cos x = 0 and y(0) = 1, then is equal to

(1) $\frac{1}{3}$ (2) $\frac{-2}{3}$ (3) $\frac{-1}{3}$ (4) $\frac{4}{3}$

Ans. [1]

Sol. $(2 + \sin x)\,\frac{dy}{dx} + (y + 1)\cos x = 0,\; y(0) = 1,\;\; y\left(\frac{\pi}{2}\right) = ?$

$$\frac{dy}{y+1} + \frac{\cos x}{2+\sin x}dx \;=\; 0$$

ln (y +1) + ln (2 + sin x) = ln C

ln {(y + 1) (2 + sin x)} = ln C

(y + 1) (2 + sin x) = C

x = 0, y = 1

$\Rightarrow$ (2) (2) = C $\Rightarrow$ C = 4.

$$y \;+\; 1 \;= \frac{4}{2+\sin x}$$

$$\text{Put}\, x = \frac{\pi}{2}$$

$$y \;+\; 1 \;= \frac{4}{2+1} = \frac{4}{3}.$$

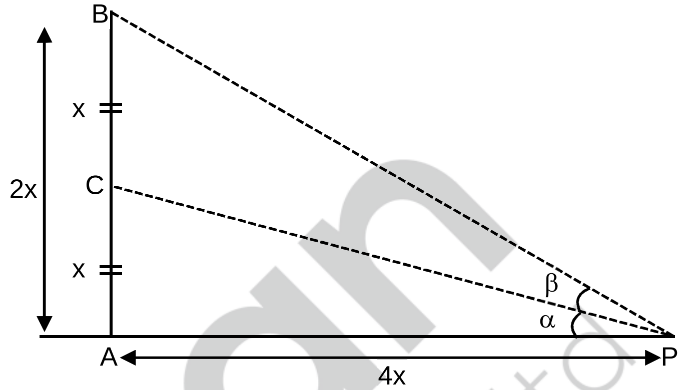
$y\left(\frac{\pi}{2}\right) = \frac{1}{3}$. **Ans.**

12. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that AP = 2AB. If $\angle BPC = \beta$, then $\tan\beta$ is equal to

(1) $\frac{6}{7}$ (2) $\frac{1}{4}$ (3) $\frac{2}{9}$ (4) $\frac{4}{9}$

Ans. [3]

Sol. $\tan\alpha = \frac{x}{4x} = \frac{1}{4}$



$\tan(\alpha+\beta) = \frac{2x}{4x} = \frac{1}{2}$

$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta} = \frac{1}{2}$

$\Rightarrow 2(\tan\alpha + \tan\beta) = 1 - \tan\alpha \cdot \tan\beta$

$\Rightarrow 2\left(\frac{1}{4} + \tan\beta\right) = 1 - \frac{\tan\beta}{4} \Rightarrow \frac{1}{2} + 2\tan\beta = 1 - \frac{\tan\beta}{4}$

$\Rightarrow 2\tan\beta + \frac{\tan\beta}{4} = \frac{1}{2} \Rightarrow \frac{9}{4} \Rightarrow \tan\beta = \frac{1}{2} \Rightarrow \tan\beta = \frac{2}{9}$. **Ans.**

13. If A = $\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then adj $(3A^2 + 12A)$ is equal to

(1) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$ (2) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$ (3) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$ (4) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

Ans. [2]

Sol. $A^2 = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix}$

$X = 3A^2 + 12A = 3\begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix} + 12\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

adj $(3A^2 + 12A)$ = adj (X) = $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$. **Ans.**

14. For any three positive real numbers a, b and c, $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$. Then

(1) b, c and a are in G.P.
(2) b, c and a are in A.P.
(3) a, b and c are in A.P.
(4) a, b and c are in G.P.

Ans. [2]

Sol. $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$

$(15a)^2 + (3b)^2 + (5c)^2 - 75ac = 45ab + 15bc$

$(15a)^2 + (3b)^2 + (5c)^2 - 45ab - 15bc - 75ac = 0$

$\because$ $a^2 + b^2 + c^2 - ab - bc - ca = 0 \Rightarrow a = b = c$

then 15a = 3b = 5c = k (let)

$a = \frac{k}{15}, b = \frac{k}{3}, c = \frac{k}{5}$

Hence, 2c = a + b.

$\therefore$ b, c and a are in A.P.

15. The distance of the point (1, 3, – 7) from the plane passing through the point (1, – 1, – 1) having normal perpendicular to both the lines $\frac{x-1}{1} = \frac{y+2}{-2} = \frac{z-4}{3}$ and = = $\frac{x-2}{2} = \frac{y+1}{-1} = \frac{z+7}{-1}$ is

(1) $\frac{20}{\sqrt{74}}$ (2) $\frac{10}{\sqrt{83}}$ (3) $\frac{5}{\sqrt{83}}$ (4) $\frac{10}{\sqrt{74}}$

Ans. [2]

Sol. Equation of plane: a (x – 1) + b (y + 1) + c (z + 1) = 0 this plane perpendicular to given two lines, then a – 2b + 3c = 0

2a – b – c = 0

$\frac{a}{5} = \frac{b}{7} = \frac{c}{3}$

then equation of plane is 5 (x – 1) + 7 (y + 1) + 3 (z + 1) = 0

5x + 7y + 3z + 5 = 0

$d = \left|\frac{5+21-21+5}{\sqrt{25+49+9}}\right| = \frac{10}{\sqrt{83}}$. **Ans.**

16. Let $I_n = \int \tan^n x\, dx$, (n > 1). If $I_4 + I_6 = a \tan^5 x + bx^5 + C$, where C is a constant of integration, then the ordered pair (a, b) is equal to

(1) $\left(\frac{-1}{5}, 1\right)$ (2) $\left(\frac{1}{5}, 0\right)$ (3) $\left(\frac{1}{5}, -1\right)$ (4) $\left(\frac{-1}{5}, 0\right)$

Ans. [2]

Sol. $I_n = \int \tan^n x\, dx$, (n > 1)

$I_4 + I_6 = a\tan^5 x + bx^5 + C$

$I_6 + \int \tan^6 x\, dx = \int \tan^4 x(\sec^2 x - 1)\, dx = \int \tan^4 x \sec^2 x\, dx - I_4$

$I_6 + I_4 = \int \tan^4 x \sec^2 x\, dx = \frac{\tan^5 x}{5} + C$

$a = \frac{1}{5}$, b = 0.

17. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directrices is x = – 4, then the equation of the normal to it at $\left(1, \frac{3}{2}\right)$ is

(1) 2y – x = 2 (2) 4x – 2y = 1 (3) 4x + 2y = 7 (4) x + 2y = 4

Ans. [2]

Sol. $e = \frac{1}{2}$

Equation of directrices $x = \frac{-a}{e} = -4 \Rightarrow 2a = 4 \Rightarrow a = 2$

$b^2 = a^2 (1 - e^2) = 4 \left(1 - \frac{1}{4}\right) = 3$

Equation of ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$

Normal at $\left(1, \frac{3}{2}\right), \frac{2x}{4} + \frac{2y}{3} \cdot \frac{dy}{dx} = 0$

$\frac{1}{2} + \left(\frac{dy}{dx}\right) = 0$

$M_T = \frac{-1}{2}$; $M_{normal} = 2$

Equation of normal $y - \frac{3}{2} = 2(x - 1)$; $2y - 3 = 4x - 4$

$\Rightarrow 4x - 2y - 1 = 0$. **Ans.**

18. A hyperbola passes through the point P $(\sqrt{2}, \sqrt{3})$ and has foci at (± 2, 0). Then the tangent to this hyperbola at P also passes through the point

(1) $(3\sqrt{2}, 2\sqrt{3})$ (2) $(2\sqrt{2}, 3\sqrt{3})$ (3) $(\sqrt{3}, \sqrt{2})$ (4) $(-\sqrt{2}, -\sqrt{3})$

Ans. [2]

Sol. Hyperbola P $(\sqrt{2}, \sqrt{3})$, foci (± 2, 0)

$\pm ae = \pm 2 \Rightarrow ae = 2$ ……. (1)

Let hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, \frac{2}{a^2} - \frac{3}{b^2} = 1$ ……. (2)

$b^2 = a^2 (e^2 - 1) \Rightarrow b^2 = (ae)^2 - a^2$

$b^2 = 4 - a^2$ …….(3)

Put in (ii)

$\frac{2}{a^2} - \frac{3}{4 - a^2} = 1$

$8 - 2a^2 - 3a^2 = a^2\,(4 - a^2)$

$8 - 5a^2 = 4a^2 - a^4$

$a^4 - 9a^2 + 8 = 0$

$a^2 = 1$, $a^2 = 8$, reject from equation (iii)

$\therefore\ a^2 = 1,\ b^2 = 3$

Hyperbola is $\dfrac{x^2}{1} - \dfrac{y^2}{3} = 1$

Tangent to the hyperbola at $P\left(\sqrt{2}, \sqrt{3}\right)$ is

$\sqrt{2}\,x - \dfrac{\sqrt{3}\,y}{3} = 1 \qquad \Rightarrow\ 3\sqrt{2}\,x - \sqrt{3}\,y = 3$

$\Rightarrow \sqrt{6}\,x - y = \sqrt{3}$

$\left(2\sqrt{2}, 3\sqrt{3}\right)$ satisfies the above tangent.

19. The function $f : R \to \left[\dfrac{-1}{2}, \dfrac{1}{2}\right]$ defined as $f(x) = \dfrac{x}{1+x^2}$, is

(1) invertible

(2) injective but not surjective

(3) surjective but not injective

(4) neither injective nor surjective

Ans. [3]

Sol. $f: R \to \left[\dfrac{-1}{2}, \dfrac{1}{2}\right]$

$f(x) = \dfrac{x}{1+x^2}$

Domain $x \in R$

$f'(x) = \dfrac{(1+x^2) - x\,(2x)}{\left(1+x^2\right)^2} = \dfrac{1-x^2}{\left(1+x^2\right)^2}$

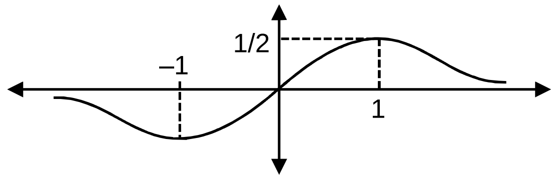
Many one

Range: $y + yx^2 = x \Rightarrow yx^2 - x + y = 0$, $D \ge 0$; $x \in R$

$1 - 4y^2 \ge 0 \Rightarrow 4y^2 - 1 \le 0 \Rightarrow y \in \left[\dfrac{-1}{2}, \dfrac{1}{2}\right]$ onto

not injective but surjective.

Graph of f (x) is



20. $\lim_{x\to\frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$ equals

(1) $\frac{1}{24}$ (2) $\frac{1}{16}$ (3) $\frac{1}{8}$ (4) $\frac{1}{4}$

Ans. [2]

Sol. $\lim_{x\to\frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$

Put $\pi - 2x = t$

$x \to \frac{\pi}{2}$, $t \to 0$; $x = \frac{\pi}{2} - \frac{t}{2}$

$$\lim_{t\to 0} \frac{\tan\frac{t}{2} - \sin\frac{t}{2}}{t^3} = \lim_{t\to 0} \tan\frac{t}{2}\left(\frac{1-\cos\frac{t}{2}}{t^3}\right) = \lim_{t\to 0} \frac{\tan\frac{t}{2}}{2\cdot\frac{t}{2}} \cdot \left(\frac{1-\cos\frac{t}{2}}{\left(\frac{t}{2}\right)^2 (2)^2}\right) = \frac{1}{16}$$

21. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. Let $\vec{c}$ be a vector such that $|\vec{c} - \vec{a}| = 3$, $\left|(\vec{a}\times\vec{b})\times\vec{c}\right| = 3$ and the angle between $\vec{c}$ and $\vec{a}\times\vec{b}$ be 30°. Then $\vec{a}\cdot\vec{c}$ is equal to

(1) $\frac{25}{8}$ (2) 2 (3) 5 (4) $\frac{1}{8}$

Ans. [2]

Sol. Given ϕ = $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$

$\vec{b} = \hat{i} + \hat{j}$

$\vec{a}\times\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$ ....(1)

$\Rightarrow \left|\vec{a}\times\vec{b}\right| = 3$

Now, $\left|(\vec{a}\times\vec{b})\times\vec{c}\right| = \left|(\vec{a}\times\vec{b})\right| |\vec{c}|$ = sin θ = 3

$\Rightarrow$ | 3 | $|\vec{c}|$ sin 30° = 3 $\Rightarrow$ $|\vec{c}| = 2$ ....(2)

Again given $\left|\vec{c} - \vec{a}\right|$ = 3, squaring both sides

$|\vec{c}|^2 + |\vec{a}|^2 - 2\,\vec{a}\cdot\vec{c} = 9$

$|2|^2 + |3|^2 - 2\,\vec{a}\cdot\vec{c} = 9 \Rightarrow 4 = 2\,\vec{a}\cdot\vec{c} \Rightarrow \vec{a}\cdot\vec{c} = 2$. **Ans.**

22. The normal to the curve y (x – 2) (x – 3) = x + 6 at the point where the curve intersects the y-axis passes through the point

(1) $\left(\frac{-1}{2}, \frac{-1}{2}\right)$ (2) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (3) $\left(\frac{1}{2}, \frac{-1}{3}\right)$ (4) $\left(\frac{1}{2}, \frac{1}{3}\right)$

Ans. [2]

Sol. Given curve y × (x – 2) (x – 3) = (x + 6) …….(1)

Let required point is P, where curve intersects y-axis.

For P, put x = 0 ⇒ y = 1, so P(0, 1)

Equation (1) differentiate w.r.t. x

$$(x-2)(x-3)\frac{dy}{dx} + y(2x-5) = 1$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(0,1)} = 1$$

Slope of normal = – 1

Equation of normal at P (0, 1) ⇒ $(y - y_1) = m(x - x_1)$

y – 1 = – 1 (x – 0) ⇒ x + y = 1 …….(2)

By option $\left(\frac{1}{2}, \frac{1}{2}\right)$ will satisfy x + y = 1.

23. If two different numbers are taken from the set {0, 1, 2, 3, ……, 10}; then the probability that their sum as well as absolute difference are both multiple of 4, is

(1) $\frac{6}{55}$ (2) $\frac{12}{55}$ (3) $\frac{14}{45}$ (4) $\frac{7}{55}$

Ans. [1]

Sol. Given set {0, 1, 2, 3, 4, 5, 6, 7, 8, 9,10}

A + Q favorable cases are: (0, 4), (0, 8), (2, 6), (2, 10), (4, 8), (6, 10)

$$p(E) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{6}{{}^{11}C_2} = \frac{6}{55}.$$ **Ans.**

24. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is

(1) 485 (2) 468 (3) 469 (4) 484

Ans. [1]

Sol. M → Men, L → Ladies

	I	II	III	IV
X	3M 0L	2M 1L	1M 2L	0M 3L
Y	3L 0M	2L 1M	1L 2M	0L 3M

X has 7 friends 3 men and 4 ladies

Y has 7 friends 4 men and 3 ladies.

Total number of ways

= 1 + 144 + 324 + 16 = 485. **Ans.**

$$\underbrace{{}^3C_3 \times {}^4C_0 \times {}^3C_3 \times {}^4C_0}_{\text{I}} + \underbrace{{}^3C_2 \times {}^4C_1 \times {}^4C_1 \times {}^3C_2}_{\text{II}} + \underbrace{{}^3C_1 \times {}^4C_2 \times {}^3C_1 \times {}^4C_2}_{\text{III}} + \underbrace{{}^3C_0 \times {}^4C_3 \times {}^3C_0 \times {}^4C_3}_{\text{IV}}$$

25. The value of $({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) + ({}^{21}C_4 - {}^{10}C_4) + \ldots\ldots + ({}^{21}C_{10} - {}^{10}C_{10})$ is

(1) $2^{21} - 2^{11}$ (2) $2^{21} - 2^{10}$ (3) $2^{20} - 2^{9}$ (4) $2^{20} - 2^{10}$

Ans. [4]

Sol. Let ${}^{21}C_1 + {}^{21}C_2 + {}^{21}C_3 + {}^{21}C_4 + \ldots\ldots + {}^{21}C_{10} = A$

${}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4 + \ldots\ldots + {}^{10}C_{10} = B$

By solving A

${}^{21}C_0 + {}^{21}C_1 + {}^{21}C_2 + {}^{21}C_4 + \ldots\ldots + {}^{21}C_{21} = 2^{21}$

$1 + {}^{21}C_1 + {}^{21}C_2 + \ldots\ldots\ {}^{21}C_{10} + {}^{21}C_{11} + {}^{21}C_{12} + \ldots\ldots + {}^{21}C_{20} + 1 = 2^{21}$

$2\,({}^{21}C_1 + {}^{21}C_2 + \ldots\ldots + {}^{21}C_{10}) = 2^{20} - 2$

${}^{21}C_1 + {}^{21}C_2 + \ldots\ldots + {}^{21}C_{10} = 2^{20} - 1$ …….(1)

Similarly and solving for B

${}^{10}C_0 + {}^{10}C_1 + \ldots\ldots + {}^{10}C_{10} = 2^{10}$

${}^{10}C_1 + {}^{10}C_2 + \ldots\ldots + {}^{10}C_{10} = 2^{20} - 1$ …….(2)

(1) – (2)

$A - B = 2^{20} - 2^{10}$.

26. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of the number of green balls drawn is

(1) $\frac{12}{5}$ (2) 6 (3) 4 (4) $\frac{6}{25}$

Ans. [1]

Sol. $P = \frac{15}{25} = \frac{3}{5}$

Hence, $q = \frac{2}{5}$

n = 10

So, $\sigma^2 = npq = \frac{3}{5} \times \frac{2}{5} \times 10 = \frac{12}{5}$. **Ans.**

27. Let a, b, c $\in$ R. If f(x) = ax^2 + bx + c is such that a + b + c = 3 and

f(x + y) = f(x) + f(y) + xy, $\forall$ x, y $\in$ R, then $\sum_{n=1}^{10} f(n)$ is equal to

(1) 330 (2) 165 (3) 190 (4) 255

Ans. [1]

Sol. Putting y = 1

f (x + 1) – f (x) = 3 + x

a(x + 1)^2 + b(x + 1) + c – ax^2 – bx – c = 3 + x

2ax + a + b = 3 + x

$\Rightarrow$ a = $\frac{1}{2}$; a + b = 3 $\Rightarrow$ b = $\frac{5}{2}$; c = 0

f (x) = $\frac{1}{2}$x^2 + $\frac{5}{2}$ x

Now, f(1) + f(2) + f(3) + f(4) + f(5) + f(6) + f(7) + f(8) + f(9) + f(10)

$$\left(\frac{1}{2}+\frac{5}{2}\right)+\left(\frac{4}{2}+\frac{10}{2}\right)+\ldots\ldots+\left(\frac{100}{2}+\frac{50}{2}\right)=\left(\frac{1}{2}+\frac{4}{2}+\frac{9}{2}+\ldots\ldots+\frac{100}{2}\right)+\left(\frac{5}{2}+\frac{10}{2}+\ldots\ldots+\frac{50}{2}\right)$$

$$= \ =\frac{1}{2}\left(\frac{10\,(10+1)(20+1)}{6}\right)+\frac{5}{2}\left[\frac{10\,(11)}{2}\right]= \ 330.$$ **Ans.**

Aliter: Given f(x) = ax^2 + bx + C

such that a + b + c = 3

and f(x + y) = f(x) + f(y) + xy, $\forall$ x $\in$ R …….(1)

Equation (1) differentiate w.r.t. x

f '(x + y) = f '(x) + y …….(2)

Equation (1) again differentiate w.r.t. y

f '(x + y) = f '(y) + x …….(3)

(2) = (3)

f '(x) + y = f '(y) + x

f '(x) – x = f '(y) – y = k

Integrate f '(x) – x = k w.r.t. x

$$\int\left(f'(x)-x\right)dx=\int k\,dx$$

f(x) = kx + $\frac{x^2}{2}$ + C …….(4)

Compare it by f(x) = ax^2 + bx + C

f(0) = C = 0; f(1) = k + $\frac{1}{2}$ + C = 3 $\Rightarrow$ k = $\frac{5}{2}$ = b

$\Rightarrow a = \frac{1}{2}, \; b = 3 - \frac{1}{2} = \frac{5}{2}, \; C = 0$

So, $f(x) = \frac{x^2}{2} + \frac{5x}{2}$

Now, f(1) + f(2) + f(3) + f(4) + f(5) + f(6) + f(7) + f(8) + f(9) + f(10)

$$\left(\frac{1}{2}+\frac{5}{2}\right)+\left(\frac{4}{2}+\frac{10}{2}\right)+\ldots\ldots+\left(\frac{100}{2}+\frac{50}{2}\right)=\left(\frac{1}{2}+\frac{4}{2}+\frac{9}{2}+\ldots\ldots+\frac{100}{2}\right)+\left(\frac{5}{2}+\frac{10}{2}+\ldots\ldots+\frac{50}{2}\right)$$

$$=\frac{1}{2}\left(\frac{10\,(10+1)(20+1)}{6}\right)+\frac{5}{2}\left[\frac{10\,(11)}{2}\right]= 330. \text{ Ans.}$$

28. The radius of a circle, having minimum area, which touches the curve $y = 4 - x^2$ and the lines, $y = |x|$ is

(1) $2\left(\sqrt{2}+1\right)$ (2) $2\left(\sqrt{2}-1\right)$ (3) $4\left(\sqrt{2}-1\right)$ (4) $4\left(\sqrt{2}+1\right)$

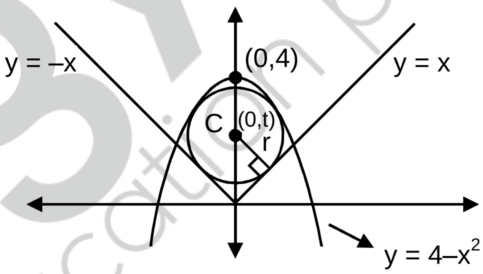
Ans. [3]

Sol. Let equation of the circle be

$x^2 + (y - t)^2 = r^2$ ......(1)

$y = 4 - x^2$ ......(2)

$t = r\sqrt{2}$ .....(3)



From (1) and (2)

$$4 - y + \left(y - r\sqrt{2}\right)^2 = r^2$$

$$y^2 - y\left(2\sqrt{2}\,r + 1\right) + r^2 + 4 = 0$$

Now, D = 0 $\Rightarrow$

$\left(2\sqrt{2}\,r + 1\right)^2 - 4\left(r^2 + 4\right) = 0$

$4r^2 + 4\sqrt{2}r - 15 = 0 \quad \Rightarrow \quad 4r^2 + 2\cdot 2\sqrt{2}r + 2 = 17$

$\Rightarrow \quad \left(2r + \sqrt{2}\right)^2 = 17 \quad \Rightarrow 2r + \sqrt{2} = \sqrt{17}$

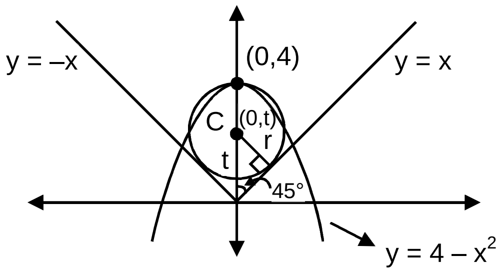
$r = \frac{\sqrt{17} - \sqrt{2}}{2} \cong 1.3$

Again, when the circle touches the parabola at (0, 4) then

$4 - t = r$ ...(i)

$t = r\sqrt{2}$ ...(ii)

from (i) and (ii)



$4 - r\sqrt{2} = r \Rightarrow \quad r = \frac{4}{\sqrt{2}+1}$

$\Rightarrow r = 4(\sqrt{2} - 1) \simeq 1.656.$

29. If, for a positive integer n, the quadratic equation,

$x(x+1) + (x+1)(x+2) + \ldots\ldots + \left(x + \overline{n-1}\right)(x+n) = 10n$

has two consecutive integral solutions, then n is equal to

(1) 12 (2) 9 (3) 10 (4) 11

Ans. [4]

Sol. $(x^2 + x) + (x^2 + 3x + 2) + (x^2 + 5x + 6) + \ldots\ldots + (x^2 + (2n-1)x + n(n-1)) = 10n$

$nx^2 + n^2x + (2 + 6 + 12 + \ldots\ldots + n(n-1)) = 10n$

Let $S = 2 + 6 + 12 + \ldots\ldots + T_{n-1}$

$S = \quad 2 + 6 + \ldots\ldots + T_{n-2} + T_{n-1}$

Subtract

$0 = 2 + 4 + 6 + \ldots\ldots + (T_n - T_{n-2}) - T_{n-1}$

$T_{n-1} = 2 + 4 + 6 \ldots\ldots + \text{upto } (n-1) \text{ terms}$

$T_{n-1} = 2(1 + 2 + 3 + \ldots\ldots (n-1) \text{ terms})$

$$T_{n-1} = \frac{2(n-1)n}{2} = n(n-1)$$

$$\sum T_{n-1} = \sum n^2 - \sum n = \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left[\frac{(2n+1)}{3} - 1\right] = \frac{n(n-1)(n+1)}{3}$$

So, $nx^2 + n^2x + \dfrac{n(n-1)(n+1)}{3} = 10n$

$x^2 + nx + \dfrac{n^2-1}{3} = 10 \Rightarrow 3h^2 + 3nx + n^2 - 1 = 30$

$\Rightarrow 3x^2 + 3nx + n^2 - 31 = 0$

$\because$ roots are consecutive number

So, $(\alpha - \beta) = 1 \Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 1 \Rightarrow (-n)^2 - 4\left(\dfrac{n^2 - 31}{3}\right) = 1$

$\Rightarrow 3n^2 - 4n^2 + 124 = 3 \Rightarrow n^2 = 121 \Rightarrow n = 11$. **Ans.**

30. The integral $\displaystyle\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equal to

(1) –2 (2) 2 (3) 4 (4) – 1

Ans. [2]

Sol. $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x} = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{2\cos^2\frac{x}{2}} = \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sec^2\frac{x}{2}\,dx = \frac{1}{2}\left[\frac{\tan\frac{x}{2}}{\frac{1}{2}}\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}} = \frac{1}{2}\left[\left(\tan\frac{3\pi}{8} - \tan\frac{\pi}{8}\right)\times 2\right]$

$= \left(\sqrt{2}+1\right) - \left(\sqrt{2}-1\right) = 2.$ **Ans.**

PART-B-PHYSICS

31. A radioactive nucleus A with a half life T, decays into a nucleus B. At t= 0, there is no nucleus B. At sometime t, the ratio of the number of B to that of A is 0.3. Then, t is given by:

(1) $t = \frac{T}{\log(1.3)}$ (2) $t = \frac{T}{2}\frac{\log 2}{\log 1.3}$ (3) $t = T\frac{\log 1.3}{\log 2}$ (4) t = T log (1.3)

Ans. [3]

Sol. $A \to B$, at t = 0, $N_B = 0$,

$N_A^1 = N_A e^{-\lambda t}$

$N_B = N_A - N_A^1$

$N_B = N_A (1 - e^{-\lambda t})$

$\Rightarrow \frac{N_B}{N_A} = 1 - e^{-\lambda t}$

$\Rightarrow 0.3 = 1 - e^{-\lambda t}$

$\Rightarrow 0.3 = 1 - e^{-\lambda t}$

$$\Rightarrow t = \frac{\ln\frac{10}{7}}{\lambda} = \frac{T\ln\frac{10}{7}}{\ln 2}$$

$$t = T\frac{\ln 1.3}{\ln 2}$$

32. The following observations were taken for determining surface tension T of water by capillary method: diameter of capillary, D = 1.25 × 10^{-2} m rise of water, h=1.45 ×10^{-2} m. Using g = 9.80 m/ s^2 and the simplified relation T = $\frac{rhg}{2}$ × 10^3 N/m, the possible error in surface tension is closest to:

(1) 10% (2) 0.15% (3) 1.5% (4) 2.4%

Ans. [3]

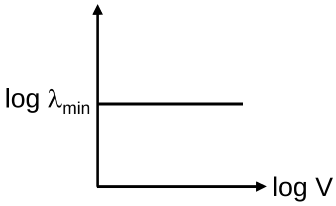
Sol. $\frac{\Delta T}{T} = \frac{\Delta r}{r} + \frac{\Delta \lambda}{\lambda}$

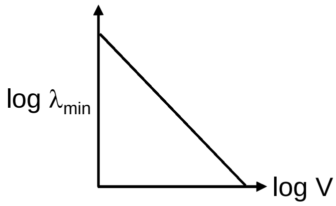
l C = 0.01, because reading is upto 2nd digit

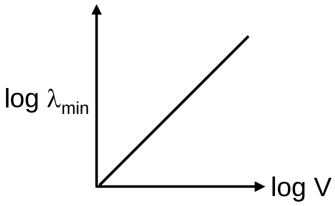
$$100 \times \left(\frac{\Delta T}{T}\right) = \left(\frac{0.01}{1.25} + \frac{0.01}{1.45}\right) \times 100$$

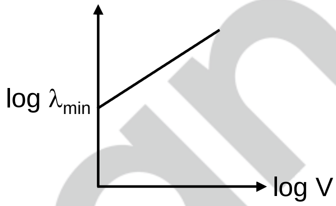
= 1.5 %

33. An electron beam is accelerated by a potential difference V to hit a metallic target to produce X-rays. It produces continuous as well as characteristic X-rays. If λ_{min} is the smallest possible wavelength of X-ray in the spectrum, the variation of log λ_{min} with log V is correctly represented in:

(A) 

(B) 

(C) 

(D) 

Ans. **[2]**

Sol. $\lambda_m = \dfrac{hc}{ev}$

$\ell n\, \lambda_m = \ell n \dfrac{hc}{ev} = \ell n\, hc - \ell n\, ev$

34. The moment of inertia of a uniform cylinder of length l and radius R about its perpendicular bisector is I. What is the ratio l/R such that the moment of inertia is minimum?

(1) $\dfrac{3}{\sqrt{2}}$ (2) $\sqrt{\dfrac{3}{2}}$ (3) $\dfrac{\sqrt{3}}{2}$ (4) 1

Ans. [2]

Sol. $I = \dfrac{m\ell^2}{12} + \dfrac{mr^2}{4}$

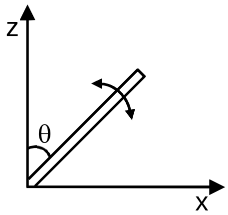
and $V = \pi r^2 \ell$

$I = \dfrac{m\ell^2}{12} + \dfrac{m}{4}\dfrac{v}{\pi\ell}$

$\dfrac{dI}{d\ell} = \dfrac{M}{12}(2l) + \dfrac{mv}{4\pi}\left(-\dfrac{1}{\ell^2}\right) = 0$

$\Rightarrow \dfrac{\ell}{r} = \dfrac{\sqrt{3}}{\sqrt{2}}$

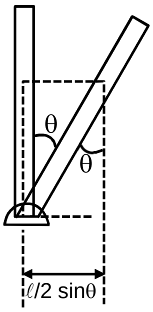
35. A slender uniform rod of mass M and length l is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is:



(A) $\frac{2g}{3\ell}\cos\theta$ (B) $\frac{3g}{2\ell}\sin\theta$ (C) $\frac{2g}{3\ell}\sin\theta$ (D) $\frac{3g}{2\ell}\cos\theta$

Ans. **[2]**

Sol.



$$\tau = mg\left(\frac{\ell}{2}\sin\theta\right) = \frac{1}{3}m\ell^2(\alpha)$$

$$\frac{3g\ell\sin\theta}{2\left(\frac{1}{3}\ell^2\right)} = \alpha$$

$$\frac{3g\sin\theta}{2\ell} = \alpha$$

36. C_p and C_v are specific heats at constant pressure and constant volume respectively. It is observed that

$C_p - C_v = a$ for hydrogen gas

$C_p - C_v = b$ for nitrogen gas

The correct relation between a and b is:

(1) a = 28 b (2) $a = \frac{1}{14}b$ (3) a = b (4) a = 14 b

Ans. [4]

Sol. $C_p - C_v = \frac{R}{m_0} = a$

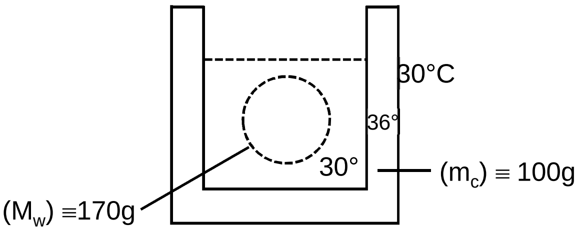
$C_p - C_v = \frac{R}{28} = b$

$\Rightarrow a = (14)\,b$

37. A copper ball of mass 100 gm is at a temperature T. It is dropped in a copper calorimeter of mass 100 gm, filled with 170 gm of water at room temperature. Subsequently, the temperature of the system is found to be 75°C. T is given by: (Given: room temperature = 30°C, specific heat of copper = 0.1 cal/gm°C)

(1) 825ºC (2) 800ºC (3) 885ºC (4) 1250ºC

Ans. [3]



m_1

T ºC $\longrightarrow$ 75º C

$\Delta Q = ms\Delta\theta$

$\Delta Q = (100)\ (0.1)\ (T - 75)$

30ºC $\longrightarrow$ 75ºC

$\Delta Q_1 = ms\ \Delta\theta$

$= (170)\ (1)\ (75 - 30)$

$\Delta Q = (170)\ (45)$

$\Delta Q_2 = ms\ \Delta\theta$

$= (100)\ (0.1)\ (75 - 30)$

$= (10)\ (45)$

$\Delta Q_2 = 450$ Cal

$100 \times 0.1\ (7 - 75) = \Delta Q_1 + \Delta Q_2$

$10T - 750 = (170 \times 45) + (450)$

$10T = (170 \times 45) + 450 + 750$

$$T = \frac{(170 \times 45) + 1200}{10}$$

$= 120 + 17 \times 45$

$= 120 + 765$

T = 885 ºC

38. In amplitude modulation, sinusoidal carrier frequency used is denoted by ω_c and the signal frequency is denoted by ω_m. The bandwidth ($\Delta\omega_m$) of the signal is such that $\Delta\omega_m << \omega_c$. Which of the following frequencies is not contained in the modulated wave?

(1) $\omega_c - \omega_m$ (2) ω_m (3) ω_c (4) $\omega_m + \omega_c$

Ans. [2]

39. The temperature of an open room of volume 30 m^3 increases from 17ºC to 27ºC due to the sunshine. The atmospheric pressure in the room remains 1×10^5 Pa. If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be:

(1) -2.5×10^{25} (2) -1.61×10^{23} (3) 1.38×10^{23} (4) 2.5×10^{25}

Ans. [1]

Sol. We know

PV = nRT

$$n_f - n_i = \frac{PV}{R}\left(\frac{1}{T_f} - \frac{1}{T_i}\right)$$

$$n_f - n_i = \frac{PV}{R}\left(\frac{1}{T_f} - \frac{1}{T_i}\right)$$

$$n_f - n_i = 6.022 \times 10^{23} \times \frac{1\times10^5 \times 30}{8.314}\left(\frac{1}{300} - \frac{1}{290}\right)$$

$n_f - n_i = -2.5 \times 10^{25}$ molecules

40. In a Young's double slit experiment, slits are separated by 0.5 mm, and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide is:

(1) 15.6 mm (2) 1.56 mm (3) 7.8 mm (4) 9.75 mm

Ans. [3]

Sol. $$\frac{n_1\lambda_1 D}{d} = \frac{n_2\lambda_2 D}{d}$$

$\Rightarrow n_1 \lambda_1 = n_2 \lambda_2$

$$\Rightarrow \frac{n_1}{n_2} = \frac{520}{650}$$

$$\Rightarrow \frac{n_1}{n_2} = \frac{4}{5}$$

$$y_1 = \frac{n_1 D\lambda_1}{d} = \frac{4\times650\times10^{-9}\times150\times10^{-2}}{0.5\times10^{-3}}$$

= 7.8 mm

41. A particle A of mass m and initial velocity v collides with a particle B of mass $\frac{m}{2}$ which is at rest. The collision is head on, and elastic. The ratio of the de-Broglie wavelengths λ_A to λ_B after the collision is:

(1) $\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$ (2) $\frac{\lambda_A}{\lambda_B} = \frac{1}{3}$ (3) $\frac{\lambda_A}{\lambda_B} = 2$ (4) $\frac{\lambda_A}{\lambda_B} = \frac{2}{3}$

Ans. [3]

Sol. $mv = mv_1 + \frac{m}{2}v_2$

$v = v_1 + \frac{v_2}{2}$ ...(1)

$v_2 - v_1 = v$ ...(2)

after sloving $v_1 = \frac{v}{3}$, $v_2 = \frac{4v}{3}$

$P_A = \frac{mv}{3}$, $P_B = \frac{m}{2}\frac{4v}{3} = \frac{2mv}{3}$

$\frac{\lambda_A}{\lambda_B} = \frac{b/P_A}{\lambda/P_B} = \frac{P_B}{P_A} = \frac{2mv}{3}\frac{3}{mv} = 2:1$

42. A magnetic needle of magnetic moment 6.7×10^{-2} Am^2 and moment of inertia 7.5×10^{-6} kg m^2 is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is

(1) 8.76 s (2) 6.65 s (3) 8.89 s (4) 6.98 s

Ans. [2]

Sol. $\alpha = \frac{BM}{I}\sin\theta$

$\cong \frac{BM}{I}\theta$

$\omega = \sqrt{\frac{BM}{I}}$

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{BM}}$

6.65×10^{-1} s

43. An electric dipole has a fixed dipole moment $\vec{p}$, which makes angle θ with respect to x-axis. When subjected to an electric field $\vec{E_1} = E\hat{i}$, it experiences a torque $\vec{T_1} = \tau\hat{k}$. When subjected to another electric field $\vec{E_2} = \sqrt{3}E_1\hat{j}$ it experiences a torque $\vec{T_2} = -\vec{T_1}$ The angle θ is

(1) 90° (2) 30° (3) 45° (4) 60°

Ans. [4]

Sol. $\vec{p}$

$\vec{E_1} = E\hat{i}$ $\quad$ $\vec{E_2} = \sqrt{3}E_1\hat{j}$

$\vec{T_1} = \tau\hat{k}$ $\quad$ $T_2 = -\vec{T_1}$

$\vec{T}_1 = \vec{P} \times \vec{E}_1$ $\qquad$ $T_2 = \vec{P} \times \vec{E}_2$

$(P\cos\theta\ \hat{i} + P\sin\theta\ \hat{j}) \times E\ \hat{i}$

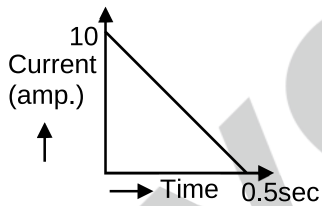
$= PE\sin\theta - \hat{k}$

$T_2 = P\,E\,\sqrt{3}\cos\theta\ \hat{k}$

Given $PE\sqrt{3}\cos\theta = PE\sin\theta$

$\tan\theta = \sqrt{3}$ $\qquad$ $\theta - 60^{o}$

44. In a coil of resistance 100 Ω, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is



(1) 275 Wb (2) 200 Wb (3) 225 Wb (4) 250 Wb

Ans. [4]

Sol. $\varepsilon = \dfrac{d\theta}{dt}$

$i = \dfrac{\varepsilon}{R}$ $\qquad$ $i = -\dfrac{10}{0.5}t + C$

i = $\qquad$ C = 10 $\qquad$ i = –20t + 10

$$\int_0^{0.5}(10-20t)R\,dt = \int_0^{\varphi} d\varphi$$

$$100\left(10\times 0.5 - 20\frac{0.25}{2}\right) = \Delta\phi$$

100 (5 – 2.5) = $\Delta\phi$, $\Delta\phi$ = 250 wb

45. A time dependent force F = 6t acts on a particle of mass 1 kg. If the particle starts from rest. the work done by the force during the first 1 sec. will be

(1) 18 J (2) 4.5 J (3) 22 J (4) 9 J

Ans. [2]

Sol. F = 6t

$\dfrac{dv}{dt} = 6t$

$$\int_0^v dv = \int_0^1 6t\,dt$$

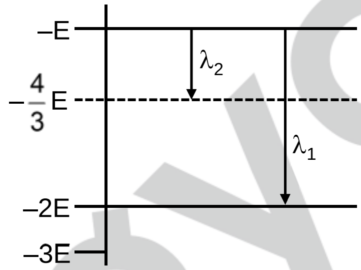
$$v = 3t^2\Big|_0^1 = 3$$

Work = chage in K.E

$$= \frac{1}{2} \times 1 \times 9$$

$$= \frac{9}{2} J$$

46. Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths $r = \frac{\lambda_1}{\lambda_2}$, is given by



(A) $r = \frac{1}{3}$ (B) $r = \frac{4}{3}$ (C) $r = \frac{2}{3}$ (D) $r = \frac{3}{4}$

Ans. **[1]**

Sol. $\Delta E_2 = \frac{1}{3}E$

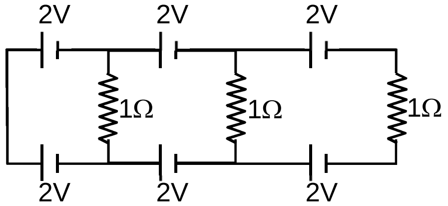
$\Delta E_1 = E$

$$\frac{\lambda_C}{\lambda_2} = \frac{1}{3}E \quad ...(1)$$

$$\frac{\lambda_C}{\lambda_1} = E \quad ...(2)$$

$$r = \frac{\lambda_1}{\lambda_2} = \frac{E/3}{E} = \frac{1}{3}$$

47.

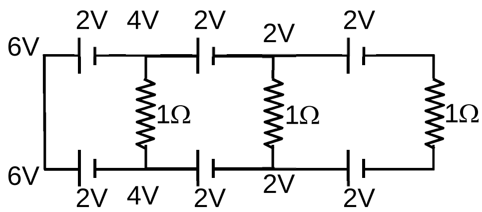


In the above circuit the current in each resistance is

(A) 0 A (B) 1 A (C) 0.25 A (D) 0.5 A

Ans. **[1]**

Sol.

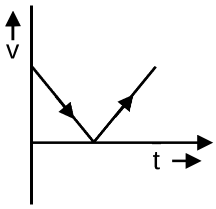


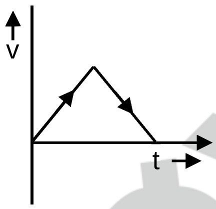
Potential difference across

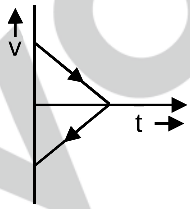
all resistors is zero

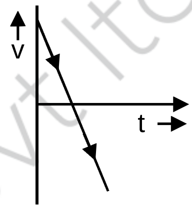
$\Rightarrow$ corrent = 0

48. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time?

(1)


(2)


(3)


(4*)


Ans. [4]

49. A capacitance of 2 μF is required in an electrical circuit across a potential difference of 1.0 kV. A large number of 1 μF capacitors are available which can withstand a potential difference of not more than 300 V. The minimum number of capacitors required to achieve this is

(1) 32 (2) 2 (3) 16 (4) 24

Ans. [1]

Sol. For the system to with ston d 1 kv potential difference

300 ×4 = 1200 V it can with stond 1 KV also

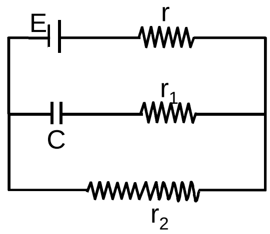
For net capacitance 2 μ F

$\frac{C}{4} = 2\ \mu\ F$

C = 8

minimum capacitors required = 8 × 4 = 32

50. In the given circuit diagram when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be



(A) CE $\left(\frac{r_1}{r_1+r}\right)$ (B) CE (C) CE $\left(\frac{r_1}{r_2+r}\right)$ (D) CE $\left(\frac{r_2}{r+r_2}\right)$

Ans. **[4]**

Sol. $i = \frac{E}{r+r_2}$

Potential difference across 'C'

= Potential difference across 'r_2'

$= \frac{E}{r+r_2} r_2$

$q = \frac{CE}{r+r_2} r_2$

51. In a common emitter amplifier circuit using an n-p-n transistor, the phase difference between the input and the output voltages will be

(1) 180° (2) 45° (3) 90° (4) 135°

Ans. [1]

52. Which of the following statements is false?

(1) Kirchhoff's second law represents energy conservation

(2) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude.

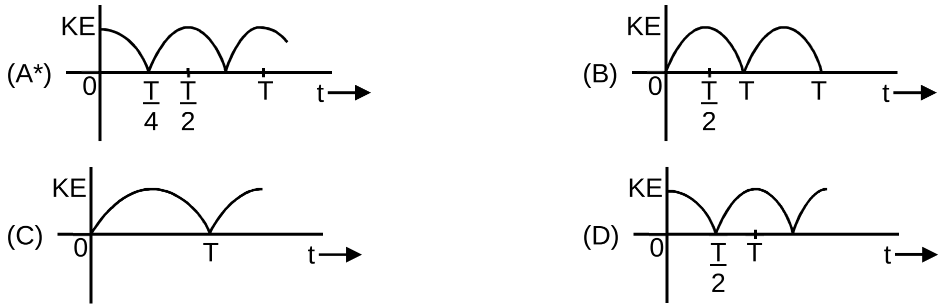
(3) In a balanced wheatstone bridge if the cell and the galvanometer are exchanged, the null point is disturbed.

(4) A rheostat can be used as a potential divider.

Ans. [3]

Sol. On changing the cell and galvanometer null point does not change

53. A particle is executing simple harmonic motion with a time period T. At time t = 0, it is at its position of equilibrium. The kinetic energy – time graph of the particle will look like



Sol. K.E. become zero at $\frac{T}{4}$

54. An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer?

(speed of light = $3 \times 10^8\ ms^{-1}$)

(1) 15.3 GHz (2) 10.1 GHz (3) 12.1 GHz (4) 17.3 GHz

Ans. [4]

Sol. Assume the abserver and source are moving away from each other with relative velocity v (v is negetive if the abserver and the source are moving towards each other)

$$\frac{f_s}{f_o} = \sqrt{\frac{1+\beta}{1-\beta}} \qquad ...(1)$$

$$\beta = \frac{v}{c} = \frac{C/2}{C} = \frac{-1}{2} \qquad ...(2)$$

Putting (2) in (1)

f_o = 17.3 GHz

55. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of

(1) $\frac{1}{81}$ (2) 9 (3) $\frac{1}{9}$ (4) 81

Ans. [2]

Sol. $\text{Stress} = \frac{F}{A_\ell} = \frac{mg}{A_\ell}$

$$S = \frac{mg}{A_\ell} = \frac{\delta_L A_B g}{A_\ell} \qquad ...(1)$$

$$S' = \frac{\delta(9\ell)(8\ell)A_B g}{81 A_\ell} \qquad ...(2)$$

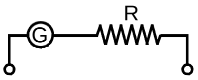
$$\frac{S}{S'} = 9$$

56. When a current of 5 mA is passed through a galvanometer having a coil of resistance 15Ω, it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into a voltmeter of range 0 – 10 V is

(1) $4.005 \times 10^3\ \Omega$ (2) $1.985 \times 10^3\ \Omega$ (3) $2.045 \times 10^3\ \Omega$ (4) $2.535 \times 10^3\ \Omega$

Ans. [2]

Sol.

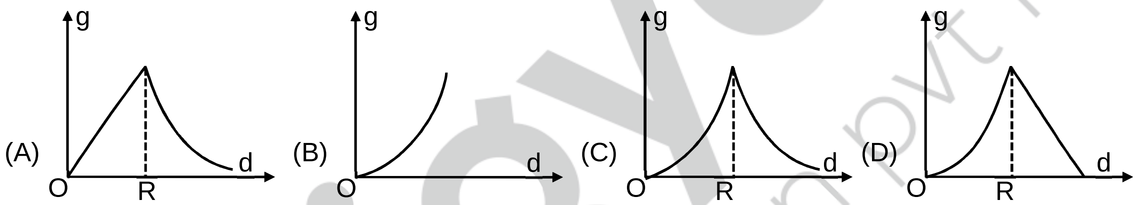


$V = i\,(R + G)$

$10 = 5 \times 10^{-3}\,(15 + R)$

$R = 2000 - 15 = 1.985 \times 10^3\ \Omega$

57. The variation of acceleration due to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius)



Ans. **[1]**

Sol. $g_{inside} = \dfrac{Gmr}{R^3}$

$g_{outside} = \dfrac{GM}{r^2}$

58. An external pressure P is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by

(1) $3PK\alpha$ (2) $\dfrac{P}{3\alpha K}$ (3) $\dfrac{P}{\alpha K}$ (4) $\dfrac{3\alpha}{PK}$

Ans. [2]

Sol. $\Delta P = -B\dfrac{\Delta V}{V}$

$\Delta V = \dfrac{-PV}{B}$, B = K (given)

and, $\Delta V = V\,(3\alpha)t$

$\dfrac{PV}{k} = V\,(3\alpha)\,t \Rightarrow t = \dfrac{P}{3ka}$

59. A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm. A beam of parallel light falls on the diverging lens. The final image formed is

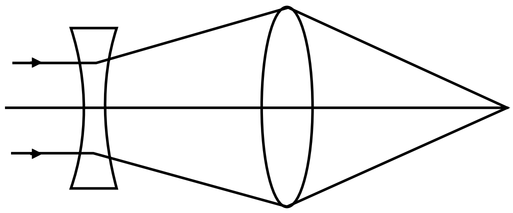
(1) real and at a distance of 6 cm from the convergent lens

(2) real and at a distance of 40 cm from convergent lens

(3) virtual and at a distance of 40 cm from convergent lens

(4) real and at a distance of 40 cm from the divergent lens.

Sol.



$V = \frac{uf}{u+f}$

$= -\frac{40 \times 20}{-40+20}$

V = 40 cm

60. A body of mass m = 10^{-2} kg is moving in a medium and experiences a frictional force F = – kv^2. Its initial speed is v_0 = 10 ms^{-1}. If, after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be

(1) 10^{-1} kg $m^{-1}s^{-1}$ (2) 10^{-3} kg m^{-1} (3) 10^{-3} kg s^{-1} (4) 10^{-4} kg m^{-1}

Ans. [4]

Sol. $m\frac{dv}{dt} = -KV^2$ and $\frac{1}{2}mv_f{}^2 = \frac{1}{8}mv_0{}^2$

$\int_{10}^{5} -m\frac{dv}{v^2} = \int_{0}^{10} +Kdg$

$K^{-2}\left(\frac{1}{5} - \frac{1}{10}\right) = 10\ K$

$K = \frac{10^{-2}}{100} = 10^{-4}\ kg\ /\ m^{-1}$

PART-C-CHEMISTRY

61. 1 gram of a carbonate (M_2CO_3) on treatment with excess HCl produces 0.01186 mole of CO_2. The molar mass of M_2CO_3 in g mol^{-1} is:

(1) 84.3 (2) 118.6 (3) 11.86 (4) 1186

Ans. [1]

Sol. $M_2CO_3 + 2HCl \longrightarrow 2MCl + H_2O + CO_2$

$n_{M_2CO_3} = n_{CO_2} = 0.01186$

0.01186 mol $\longrightarrow$ 1 gm

1 mol $\longrightarrow \left(\frac{1}{0.01186} \times 1\right)$

= 84.3 g mol^{-1}

62. Given

$C_{(graphite)} + O_2(g) \rightarrow CO_2(g);$

$\Delta_r H^o = -393.5$ kJ mol^{-1}

$H_2(g)\ \frac{1}{2} + O_2(g) \rightarrow H_2O(l);$

$\Delta_r H^o = -285.8$ kJ mol^{-1};

$CO_2(g) + 2H_2O(l) \rightarrow CH_4(g) + 2O_2(g);$

$\Delta_r H^o = +890.3$ kJ mol^{-1}

Based on the above thermochemical equations, the value of $\Delta_r H^o$ at 298 K for the reaction

$C_{(graphite)} + 2H_2(g) \rightarrow CH_4(g)$ will be:

(1) +144.0 kJ mol^{-1} (2) –74.8 kJ mol^{-1} (3) –144.0 kJ mol^{-1} (4) +74.8 kJ mol^{-1}

Ans. [2]

Sol. (1) $C_{(graphite)} + O_2(g) \longrightarrow CO_2(g)$, $\Delta_r H^o = -393.5$ kJ mol^{-1}

(2) $H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(l)$, $\Delta_r H^o = -285.8$ kJ mol^{-1}

(3) $CO_2(g) + 2H_2O(l) \longrightarrow CH_4(g) + 2O_2(g)$, $\Delta_r H^o = +890.3$ kJ mol^{-1}

$C_{(graphite)} + 2H_2(g) \rightarrow CH_4(g)$

$\Delta_r H^o = -393.5 - (2 \times 285.8) + 890.3$

$= -74.8$ kJ/mol^{-1}

63. The freezing point of benzene decreases by 0.45°C when 0.2 g of acetic acid is added to 20 g of benzene. If acetic acid associates to form a dimer in benzene, percentage association of acetic acid in benzene will be: (K_f for benzene = 5.12 K kg mol^{-1})

(1) 80.4% (2) 74.6% (3) 94.6% (4) 64.6%

Ans. [3]

Sol. $\Delta T_f = 0.45$ $CH_3COOH = 0.2$ gm $C_6H_6 = 20$ gm

$$m = \frac{0.2/60}{20} \times 1000 = \frac{1}{6}$$

$$\Delta T_f = i\,K_f \times m = 0.45 = i \times 5.12 \times \frac{1}{6}$$

$i = 0.527$

$$i = 1 - \frac{\alpha}{2}$$

$$0.527 = 1 - \frac{\alpha}{2}$$

$$\frac{\alpha}{2} = 0.473$$

$\alpha = 94.6$ %

64. The most abundant elements by mass in the body of a healthy human adult are: Oxygen (61.4%); Carbon (22.9%), Hydrogen (10.0%); and Nitrogen (2.6%). The weight which a 75 kg person would gain if all ^1H atoms are replaced by ^2H atom is:

(1) 37.5 kg (2) 7.5 kg (3) 10 kg (4) 15 kg

Ans. [2]

Sol. Gain in weight = $\left(\frac{75 \times 10}{100} \times (M_{H^2} - M_{H^1})\right)$ = 7.5 kg

65. ΔU is equal to-

(1) Isobaric work (2) Adiabatic work (3) Isothermal work (4) Isochoric work

Ans. [2]

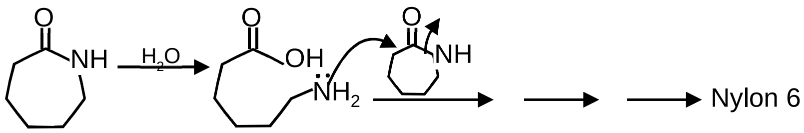
Sol. $q = 0$, $\Delta U = w$,

In adiabatic process

66. The formation of which of the following polymers involves hydrolysis reaction?

(1) Bakelite (2) Nylon 6, 6 (3) Terylene (4) Nylon 6

Ans. [4]

Sol. 

Caprolactam

67. Given

$E^{\circ}_{Cl_2/Cl^-} = 1.36V, E^{\circ}_{Cr^{3+}/Cr} = -0.74V$

$E^{\circ}_{Cr_2O_7^{2-}/Cr^{3+}} = 1.33V, E^{\circ}_{MnO_4^-/Mn^{2+}} = 1.51V$

Among the following, the strongest reducing agent is:

(1) Mn^{2+} (2) Cr^{3+} (3) Cl^- (4) Cr

Ans. [4]

Sol. Strongest Reducing agent $\rightarrow$ which undergoes oxidations

$Mn^{+2} \longrightarrow MnO_4^- + 5e^-$ $E^o_{Mn^{+2}/MnO_4^-} = -1.51V$

$2Cr^{3+} \longrightarrow Cr_2O_7^{2-} + 6e^-$ $E^o_{Cr^{+2}/Cr_2O_7^{2-}} = -1.33V$

$2Cl^- \longrightarrow Cl_2 + 2e^-$ $E^o_{Cl^-/Cl_2} = -1.36V$

$Cr \longrightarrow Cr^{+3} + 3e^-$ $E^o_{Cr/Cr^{+3}} = 0.74V$

E° (+ve) or ΔG (–ve) represents spontaneous process

68. The Tyndall effect is observed only when following conditions are satisfied:

(a) The diameter of the dispersed particles is much smaller than the wavelength of the light used

(b) The diameter of the dispersed particle is not much smaller than the wavelength of the light used

(c) The refractive indices of the dispersed phase and dispersion medium are almost similar in magnitude

(d) The refractive indices of the dispersed phase and dispersion medium differ greatly in magnitude

(1) (b) and (d) (2) (a) and (c) (3) (b) and (c) (4) (a) and (d)

Ans. [1]

69. In the following reactions, ZnO is respectively acting as a/an:

(a) $ZnO + Na_2O \rightarrow Na_2ZnO_2$

(b) $ZnO + CO_2 \rightarrow ZnCO_3$

(1) base and base (2) acid and acid (3) acid and base (4) base and acid

Ans. [3]

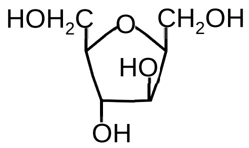
Sol. $ZnO + Na_2O \longrightarrow Na_2ZnO_2$

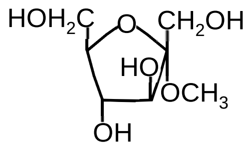
(acid) (base) (salt)

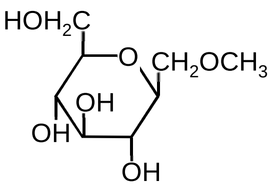
$ZnO + CO_2 \longrightarrow ZnCO_3$

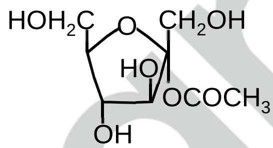
(base) (acid) (Salt)

70. Which of the following compounds will behave as a reducing sugar in an aqueous KOH solution?

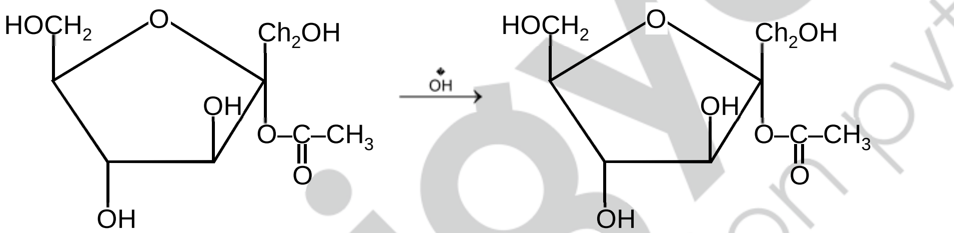
(1) 

(2) 

(3) 

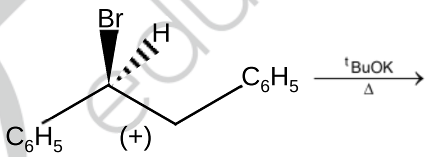
(4) 

Ans. [4]

Sol. 

(Hemeacetal linkage is present
So that reducing sugar)

71. The major product obtained in the following reaction is:



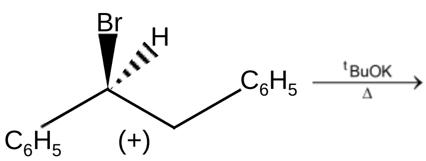
(1) $C_6H_5CH = CHC_6H_5$

(2) (+) $C_6H_5CH\ (O^tBu)\ CH_2C_6H_5$

(3) (–) $C_6H_5CH(O^tBu)\ CH_2C_6H_5$

(4) (±) $C_6H_5CH(O^tBu)\ CH_2C_6H_5$

Ans. [1]

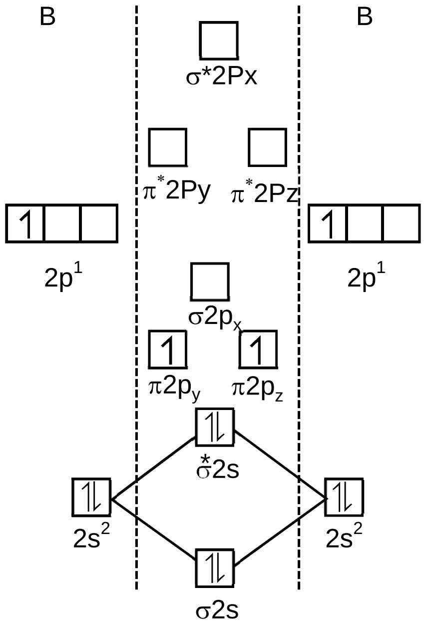
Sol.  Ph – CH = Ch – Ph

72. Which of the following species is not paramagnetic?

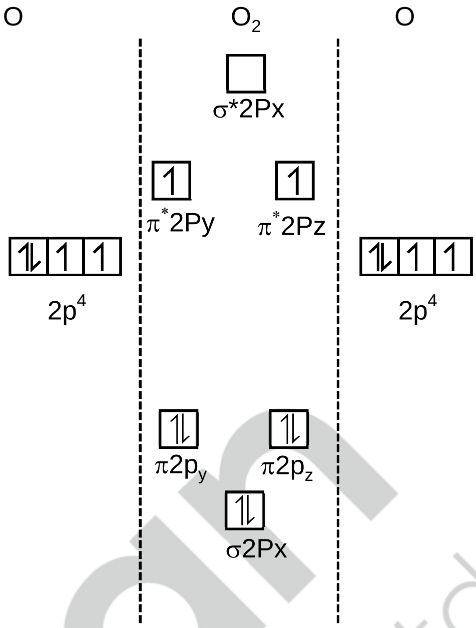
(1) CO (2) O_2 (3) B_2 (4) NO

Ans. [1]

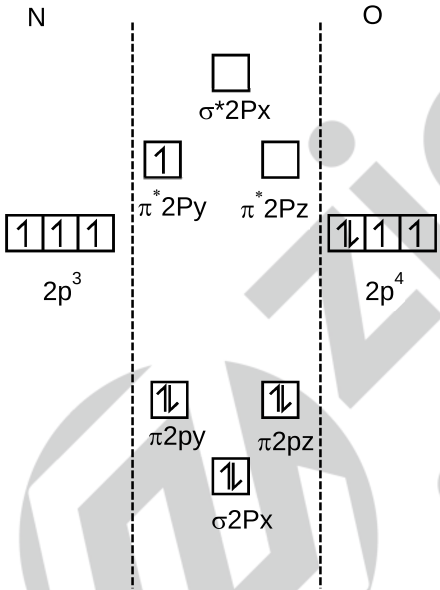
Sol.



Unpaired electron (n) = 2
(paramagnetic)



Unpaired electrons (n) = 2
(paramagnetic)



unpaired electron (n) = 1
(Paramagnetic)

73. On treatment of 100 mL of 0.1 M solution of $CoCl_3.6H_2O$ with excess $AgNO_3$; 1.2×10^{22} ions are precipitated. The complex is:

(1) $[Co(H_2O)_3Cl_3].3H_2O$

(2) $[Co(H_2O)_6]Cl_3$

(3) $[Co(H_2O)_5Cl]Cl_2.H_2O$

(4) $[Co(H_2O)_4Cl_2]\ Cl.2H_2O$

Ans. [4]

Sol. $n_{COCl_3.6H_2O} = 0.01\text{moles}$

number of moles of ions = $\dfrac{1.2\times10^{22}}{6\times10^{23}}$ = 0.02

Two mole of Ag^{+} and Cl^{-} ions are produced so

Complex is = $[Co(H_2O)_4Cl_2]Cl.2H_2O$

[0.01 mol of above complex will give 0.01 moles of Cl^{-} ions.

74. pK_a of a weak acid (HA) and pK_b of a weak base (BOH) are 3.2 and 3.4, respectively. The pH of their salt (AB) solution is:

(1) 6.9 (2) 7.0 (3) 1.0 (4) 7.2

Ans. [1]

Sol. pH = $\frac{1}{2}$ $(pK_w + pK_a - pK_b)$

= $\frac{1}{2}$(14 + 3.2– 3.4) = 6.9

75. The increasing order of the reactivity of the following halides for the S_N1 reaction is:

$CH_3CH(Cl)CH_2CH_3$ (I)

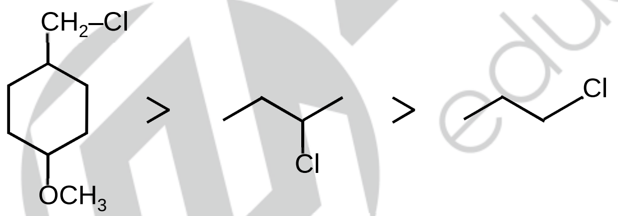
$CH_3CH_2CH_2Cl$ (II)

p–$H_3CO–C_6H_4$ – CH_2Cl (III)

(1) (II) < (I) < (III) (2) (I) < (III) < (II) (3) (II) < (III) < (I) (4) (III) < (II) < (I)

Ans. [1]

Sol.



S_N1 reactivity order

76. Both lithium and magnesium display several similar properties due to the diagonal relationship; however, the one which is incorrect is-

(1) both form soluble bicarbonates

(2) both form nitrides

(3) nitrates of both Li and Mg yield NO_2 and O_2 on heating

(4) both form basic carbonates

Ans. [4]

Sol. $LiHCO_3$ & $Mg(HCO_3)_2$ both are soluble

$6Li + N_2 \longrightarrow 2Li_3N$

$3Mg + N_2 \longrightarrow Mg_3N_2$

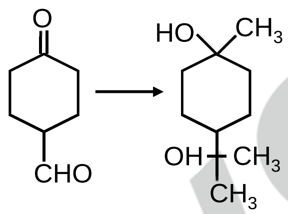
$2LiNO_3 \xrightarrow{\Delta} Li_2O + NO_2 + \frac{1}{2} O_2$

$Mg(NO_3)_2 \xrightarrow{\Delta} MgO + 2NO_2 + O_2$

Mg forms basic carbonate having formula $4MgCO_3 . Mg(OH)_2 . 6H_2O$ (white ppt.)

where as Li does not

77. The correct sequence of reagents for the following conversion will be:



(1) CH_3MgBr, H^+/CH_3OH, $[Ag(NH_3)_2]^+ OH^-$

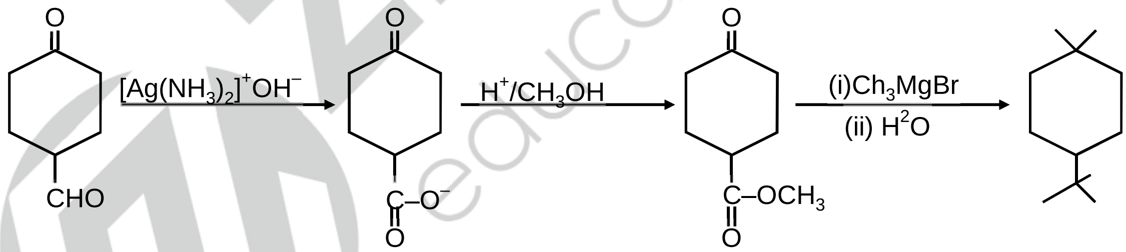
(2) CH_3MgBr, $[Ag(NH_3)_2]^+ OH^-$, H^+/CH_3OH

(3) $[Ag(NH_3)_2]^+ OH^-$, CH_3MgBr, H^+/CH_3OH

(4) $[Ag(NH_3)_2]^+ OH^-$, H^+/CH_3OH, CH_3MgBr

Ans. [4]

Sol.



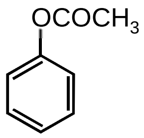
78. The products obtained when chlorine gas reacts with cold and dilute aqueous NaOH are:

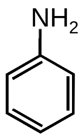
(1) ClO_2^- and ClO_3^- (2) Cl^- and ClO^- (3) Cl^- and ClO_2^- (4) ClO^- and ClO_3^-

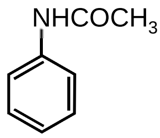
Ans. [2]

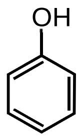
Sol. $2NaOH + \overset{0}{Cl_2} \xrightarrow[\text{dilute}]{\text{cold \&}} Na\overset{-1}{Cl} + Na\overset{+1}{Cl}O + H_2O$

79. Which of the following compounds will form significant amount of meta product during mono-nitration reaction?

(1) 

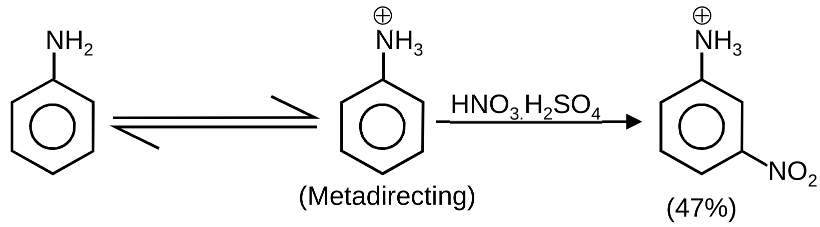
(2) 

(3) 

(4) 

Ans. [2]

Sol.

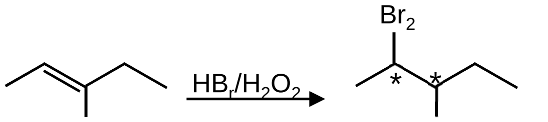


80. 3-Methyl-pent-2-ene on reaction with HBr in presence of peroxide forms an addition product. The number of possible stereoisomers for the product is:

(1) Zero (2) Two (3) Four (4) Six

Ans. [3]

Sol.



81. Two reactions R_1 and R_2 have identical pre-exponential factors. Activation energy of R_1 exceeds that of R_2 by 10 kJ mol^{-1}. If k_1 and k_2 are rate constants for reactions R_1 and R_2 respectively at 300 K, then ln (k_2/k_1) is equal to : (R = 8.314 J mol^{-1} K^{-1})

(1) 12 (2) 6 (3) 4 (4) 8Ans.

Ans. [3]

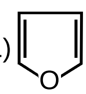
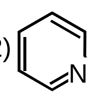
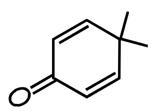
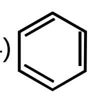
Sol. Given $[E_{a_1} - E_{a_2} = 10]$ $\quad K_1 = A.e^{-E_{a_1}/RT}$

$$\frac{K_2}{K_1} = e^{-(E_{a_2} - E_{a_1})/RT} \qquad K_2 = A.e^{-E_{a_2}/RT}$$

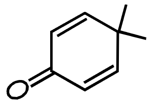
$$\ln\left(\frac{K_2}{K_1}\right) = -E_{a_2} + E_{a_1} / RT$$

$$= \left(\frac{10 \times 1000}{8.314 \times 300}\right) = 4$$

82. Which of the following molecules is least resonance stabilized?

(1)  (2)  (3)  (4) 

Ans. [3]

Sol. 

Non aromatic compound. least stablized amongst given compounds

83. The group having isoelectronic species is:

(1) O^{-}, F^{-}, Na, Mg^{+} (2) O^{2-}, F^{-}, Na, Mg^{2+} (3) O^{-}, F^{-}, Na^{+}, Mg^{2+} (4) O^{2-}, F^{-}, Na^{+}, Mg^{2+}

Ans. [4]

Sol. Which has same total number of electrons known as isoelectronic series.

O^{2-}, F^{-}, Na^{+}, Mg^{2+}

84. The radius of the second Bohr orbit for hydrogen atoms is:

(Planck's Const. h = 6.6262×10^{-34} Js;

mass of electron = 9.1091×10^{-31} kg;

charge of electron e = 1.60210×10^{-19} C;

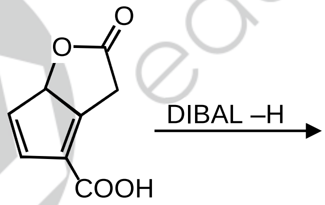
permittivity of vacuum $\epsilon_0 = 8.854185 \times 10^{-12}$ kg^{-1} m^{-3} A^2)

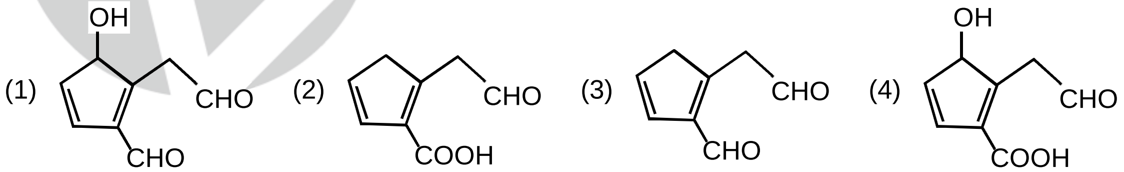
(1) 4.76Å (2) 0.529 Å (3) 2.12 Å (4) 1.65 Å

Ans. [3]

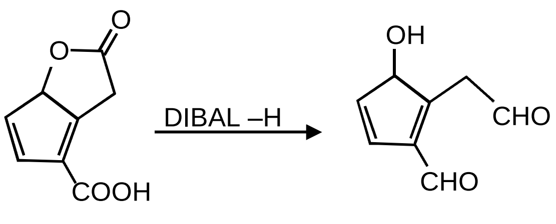
Sol. $r = 0.529 \times \frac{2^2}{1} = 2.12$ Å

85. The major product obtained in the following reaction is:





Ans. [4]

Sol. 

DIBAL-H reduces ester and carboxylic to corresponding aldehyde.

86. Which of the following reactions is an example of a redox reaction?

(1) $XeF_2 + PF_5 \rightarrow [XeF]^+ PF_6^-$ (2) $XeF_6 + H_2O \rightarrow XeOF_4 + 2HF$

(3) $XeF_6 + 2H_2O \rightarrow XeO_2F_2 + 4HF$ (4) $XeF_4 + O_2F_2 \rightarrow XeF_6 + O_2$

Ans. [4]

Sol. Redox reaction is

$$\overset{+4}{Xe}F_4 + \overset{+1}{O_2}F_2 \rightarrow \overset{+6}{Xe}F_6 + \overset{0}{O_2}$$

87. A metal crystallises in a face centred cubic structure. If the edge length of its unit cell is 'a'. the closest approach between two atoms in metallic crystal will be:

(1) $2\sqrt{2}a$ (2) $\sqrt{2}a$ 3) $\frac{a}{\sqrt{2}}$ (4) 2a

Ans. [3]

Sol. Closest approach between two atoms = 2r

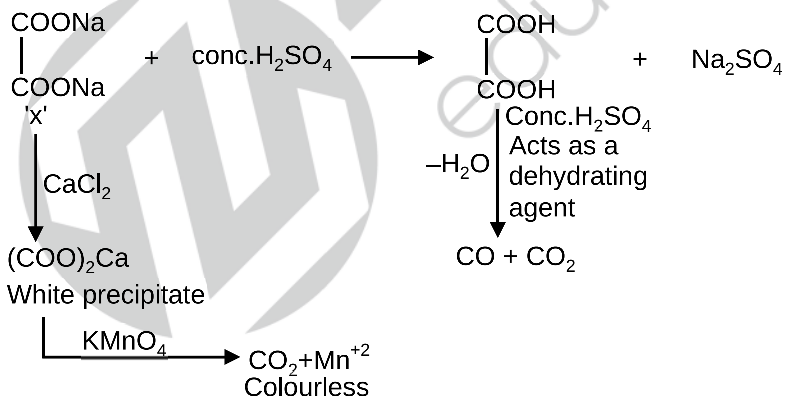
$$= \frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}}$$

88. Sodium salt of an organic acid 'X' produces effervescence with conc. H_2SO_4, 'X' reacts with the acidified aqueous $CaCl_2$ solution to give a white precipitate which decolourises acidic solution of $KMnO_4$. 'X' is:

(1) HCOONa (2) CH_3COONa (3) $Na_2C_2O_4$ (4) C_6H_5COONa

Ans. [3]

Sol.



89. A water sample has ppm level concentration of following anions

$F^- = 10$; $SO_4^{2-} = 100$; $NO_3^- = 50$

The anion/anions that make/makes the water sample unsuitable for drinking is/are-

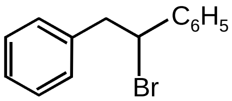
(1) both SO_4^{2-} and NO_3^- (2) only F^-

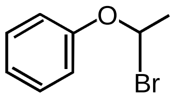
(3) only SO_4^{2-} (4) only NO_3^-

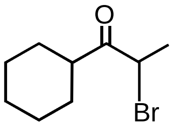
Ans. [2]

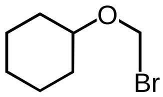
Sol. F^- ion concentration above 2 ppm causes brown mottling of teeth.

90. Which of the following, upon treatment with tert-BuONa followed by addition of bromine water, fails to decolourize the colour of bromine?

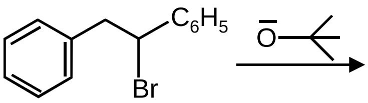
(1) 

(2) 

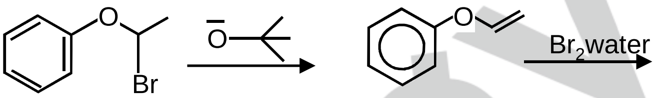
(3) 

(4) 

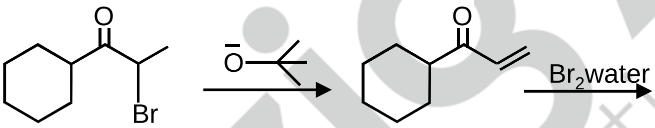
Ans. [4]

Sol. (1) 

Ph – Ch = CH – Ph $\xrightarrow{Br_2water}$ + ve test

(2) 

+ ve test

(3) 

+ ve test

(4)